Understanding Housing Wealth Effects: Debt, Home Ownership and the Lifecycle*

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Preliminary and incomplete.

Abstract

Housing wealth effects—the reaction of consumption to changes in house prices—were at the heart of the Great Recession. Empirical and quantitative macroeconomic studies have found that housing wealth effects are stronger for more indebted households. One important policy implication is that lowering debt limits for borrowers will dampen the consumption slump in a house price bust. Such conclusions might be premature. We build a simple life-cycle model with housing with closed form solutions for housing wealth effects. We show that the strength of housing wealth effects crucially depends on the underlying household characteristics which also determine the debt levels. In this framework imposing one-size-fits-all debt limits does not necessarily mitigate housing wealth effects. To be effective, policies have to be tailored to borrowers' characteristics. Aggregate housing wealth effects can be reduced in three ways: (*i*) if old homeowners reduce their housing wealth; (*ii*) if the home ownership rate decreases; (*iii*) if agents have smaller houses. We provide a simple empirical test of our model predictions. When explaining housing wealth effects, we find that the level of mortgages turns statistically insignificant once relevant household characteristics (age and a proxy for housing preferences) are added.

Keywords: Housing wealth effects, homeownership rate, house price crash, housing boom and bust, consumption dynamics, Great Recession

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1 Introduction

During the Great Recession, the US saw a pronounced drop in house prices along with a stark reduction in consumption expenditures. The large reduction

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in spending has been attributed to housing wealth effects: Households reduced their non-durable consumption as a reaction to the depreciation of their housing wealth. Mian, Rao, and Sufi (2013) emphasize the role of debt for housing wealth effects. They find that aggregate housing wealth effects are stronger in more indebted regions.[1] This finding suggests that imposing low enough debt limits is a potent policy to dampen consumption response to a house price bust. In response, as of 2018, 60% of advanced countries have introduced maximum loan-to-value ratios.[2]

In this paper we show that this conclusion might be premature. A one-size-fits-all approach to regulating debt limits might not be the best measure to increase resilience to a house price bust. Instead, measures should be taylored to household characteristics and take into account the aggregate taste for housing.

To this end, we build a simple life-cycle permanent-income model that allows closed form solutions for housing wealth effects. Housing wealth effects do not directly depend on debt. They do depend on household characteristics (age and the utility weight for housing) which also determine the level of indebtedness. Thus, there is a reduced form correlation between aggregate debt and the aggregate consumption reaction on a regional level. The sign of this correlation depends on the composition of agents.

Consider a baseline region A with a continuum of agents and compare it to three other regions. Region B has identical distributions of age and incomes, but agents have a larger utility weight of housing. Region B will be more indebted than region A and will have *stronger* housing wealth effects.

Region C has identical distributions of incomes and housing preferences, but agents are younger than in region A. Region C will be more indebted than region A and will have *weaker* housing wealth effects.

Region D has identical distributions of age and housing preferences, but agents have more front-loaded incomes than in region A. Region D will be more indebted than region A and will have *as strong* housing wealth effects as region A.

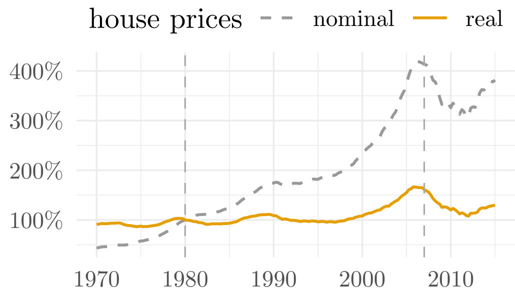
Thus, the underlying reason for being indebted changes the sign of the effect on housing wealth effects. This finding is not inconsistent with Mian et al. (2013), who find that more indebted regions had a stronger consumption response in the crisis. If the age distribution is similar across regions, their estimate is simply picking up the effect of differences in the taste for housing.

We test the predictions of our model empirically. We use data on consumption and housing from the Consumer Expenditure Survey (CEX) and MSA-level house price indices from `Zillow.com`. We construct a proxy for housing preferences from the residual of an auxiliary regression and show that mortgages are a statistically significant predictor of the size of housing wealth effects only if age and housing

[1] Aladangady (2017) provide further empirical evidence on the relationship between debt and housing wealth effects. Macroeconomic models of housing generate a similar correlation (e.g. Berger, Guerrieri, Lorenzoni, and Vavra, 2018).

[2] For more details see Cerutti, Claessens, and Laeven (2017) and `https://voxeu.org/article/increasing-faith-macroprudential-policies`.

Figure 1: Real and nominal house prices in the USA.



Notes: Nominal: Case-Shiller Home Price Index. Real: Deflated by the Consumer Price Index. Source: http://www.econ.yale.edu/~shiller/data.htm

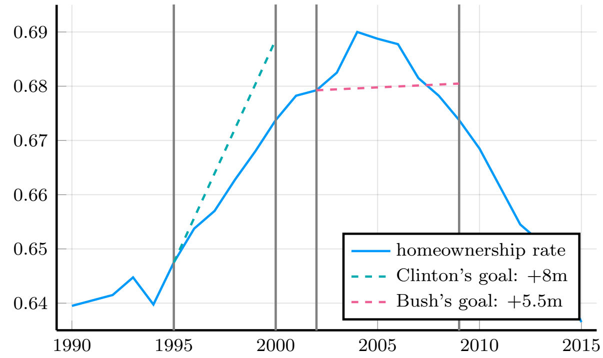
prefences are excluded from the regression. If, on the other hand, mortgages are excluded, age and housing preferences both have the predicted statistically significant positive effect on housing wealth effects.

From these findings we derive two main policy implications. First, debt limits should vary with age. For young households home ownership provides an opportunity to build up wealth. At the same time, this group does not exhibit strong housing wealth effects. Hence, it does not appear reasonable to further constrain their choices with respect LTV limits. Instead, the ability of older households to take out debt should be curtailed more strongly. It is these people that display stronger consumption reactions and thereby impose more of the negative externalities of housing wealth effects on the economy. The reason is that housing makes up a larger share of older households' remaining lifetime wealth, therefore they react stronger to price busts. Hence, from the policy maker's perspective it makes sense to encourage diversification of senior people's portfolios into other assets such as stocks and downsizing of houses. The importance of this conclusion will only increase during the coming years when the baby boomer generation, which holds a larger share of national wealth (in housing) is about to enter retirement. According to our model, a given housing bust in the future will lead to a more severe reduction in consumption expenditures compared to today due to the changing demographics.

Second, politicians should reconsider policies which promote home ownership. In the US such policies often convey the message that owning a big house is still part of the American Dream. Through the lense of our model, a high home ownership rate can be considered as an expression of strong household preferences for housing. However, stronger preferences for housing are associated with bigger housing wealth effects. Hence, policies that raise home ownership contribute to financial instability.

Contributions to the literature This paper connects to the empirical and the quantitative macroeconomic literatures on housing wealth effects. We provide

Figure 2: Homeownership in the US and policy goals.



a tractable model that rationalizes both empirical and quantitative findings about the heterogeneity of housing wealth effects.

Empirical studies have shown that the reaction of consumption to changes in housing wealth varies with household characteristics. Mian et al. (2013) show that counties that are more leveraged and poorer react more strongly to house price changes. Campbell and Cocco (2007) find heterogeneous housing wealth effects with respect to housing status and age. They find strongest reactions for older homeowners and the weakest reaction for young renters. Aladangady (2017) shows that the response to house prices is neglibile for renters and large for homeowners. His specifications implies that the consumption response is proportional to initial house values.

Our model generates the findings that homeowners that are older or have bigger houses react stronger while renters don't react. We show that the role of indebtedness is ambiguous on an individual level, but likely negative (consistent with previous findings) on an aggregate level.

Similar to the empirical studies mentioned above, quantitative macroeconomic studies have found that housing wealth effects vary across individuals. Guren, McKay, Nakamura, and Steinsson (2020) show that the consumption response, as a function of loan-to-value (LTV), is hump shaped. Berger et al. (2018) show numerically that consumption elasticities vary with income, age, housing, liquid assets and renting decision. With the exception of liquid assets, we can investigate all of these dimensions analytically in our model.

Moreover, Berger et al. (2018) derive a rule-of-thumb for housing wealth effects, which are given by the initial value of the house times the marginal propensity to consume. We complement their finding by providing a formula that is solely based on primitives of the model, without relying on an endogenous object like the MPC.

Structure of the paper The paper is organized as follows. In Section 2 we present our tractable life-cyle model with housing and mortgages and its solution. Subsequently, we derive closed forms for housing wealth effects in our model (this is our main result) and discuss comparative statics in Section 3. In Section 4 we provide an empirical test of our model's predictions.

2 A Simple Lifecycle Model with Housing

Time is discrete and runs forever. Households are born with an initial endowment of assets $a_0 \geq 0$ and live for $J \in \mathbb{N}$ periods. There are two types of households: homeowners and renters. These two types differ in their access to technology: homeowners are not allowed to rent, renters are not allowed to buy.[3] Households derive utility from a non-durable consumption good c and their durable housing stock h (rented or owned). They supply labor inelastically and receive earnings y. Households choose streams of consumption $c_t > 0$, housing stock $h_t > 0$ and assets $a_t \in \mathbb{R}$ to maximize their discounted lifetime utility.

2.1 Homeowners

Homeowners' discounted lifetime utility is

$$\sum_{t=0}^{J-1} \beta^t \frac{\left(c_t^{1-\xi} h_t^{\xi}\right)^{1-\gamma}}{1-\gamma} + \beta^{J-1}\psi(h_{J-1}),$$

where $\beta > 0$ is the discount factor and $\psi : \mathbb{R}_+ \to \mathbb{R}$ represents a warm-glow bequest motive. Households consume a Cobb-Douglas-aggregated composite good from which they derive utility according to a standard constant relative risk aversion (CRRA) utility function.

Housing is both a consumption good and an asset. It is modelled as a homogeneous, divisible good. As such, h represents a one-dimensional measure of housing quality (including size, location and amenities). An agent's housing stock depreciates at rate δ and can be adjusted frictionlessly. Home improvements and maintenance expenditures x_t have the same price as housing (p) and go into the value of the housing stock one for one. The law of motion for the housing stock is

$$h_t = (1-\delta)h_{t-1} + x_t,$$

where $h_{-1} = 0$. The asset a serves both as a savings device and short-term mortgage. Saving and borrowing can be done at the equilibrium interest rate r. The law of motion for end-of-period assets is

$$a_{t+1} = (1+r)a_t + y_t - c_t - px_t,$$

where a_0 is the given initial endowment. Agents are not allowed to die in debt,

$$a_{J+1} \geq 0.$$

[3] This assumption is a short-cut to explicitly modelling the renting-vs-owning decision of households.

Otherwise, there are no borrowing constraints in our model. This is justified, because there is no uncertainty, and thus, no reason to default in our model.

In order to obtain closed-form results for optimal choices, we make the following assumptions. First, we assume that incomes are deterministic and constant over time.

Assumption 1 (Constant incomes)**.** $y_t = y$ for all t.

Second, we assume that the bequest function takes the following parametric form.

Assumption 2. The bequest function is $\psi(h,p) = \kappa_1 \frac{(\kappa_2 h)^{1-\gamma}}{1-\gamma}$ where $\kappa_1 = \xi \frac{\beta(1-\delta)}{1-\beta(1-\delta)}$ and $\kappa_2 = \big((1-\beta(1-\delta))\frac{1-\xi}{\xi}p\big)^{1-\xi}$.

Assumption 2 will ensure that optimal choices for consumption c and housing stock h are constant over time. It requires the marginal utility of bequeathing a house being equal to the marginal utility of selling the house during one's lifetime.

Optimal choices

We first show that the optimal consumption and housing stock for a household is constant over time.

Lemma 1. *Under Assumptions 1 and 2 the optimal choices for housing stock and consumption are constant over time, $c_t = c$ and $h_t = h$, for all $0 < t < J-1$ and in optimum $c(h) = \kappa_3 ph$, where $\kappa_3 = (1-\beta(1-\delta))\frac{1-\xi}{\xi}$.*

Proof. See Appendix A.1. □

Then we derive optimal choices for consumption c and housing h.

Proposition 1. *Under Assumptions 1 and 2 optimal choices are*

$$\begin{aligned} ph &= \mathcal{Y}\frac{1}{1-\delta+\theta(J,r)\cdot(\delta+\kappa_3)}, \\ c &= \kappa_3 ph, \end{aligned}$$

where $\theta(J,r) := \sum_{j=0}^{J-1}\left(\frac{1}{1+r}\right)^j$ and $\mathcal{Y}$ is life-time income.

Proof. The life-time budget constraint is given by

$$(1-\delta)ph + \sum_{j=0}^{J-1}\Big(\frac{1}{1+r}\Big)^j(\delta ph + c) = a_0 + \sum_{j=0}^{J-1}\Big(\frac{1}{1+r}\Big)^j y =: \mathcal{Y}$$

Using the definition of θ, and the assumption on c we get

$$\begin{aligned} \mathcal{Y} &= (1-\delta)ph + \theta(J,r)\cdot(\delta ph + \kappa_3 ph) \\ &= ph\big((1-\delta) + \theta(J,r)\cdot(\delta+\kappa_3)\big) \end{aligned}$$

Rearranging yields the desired result. □

(a) Infinite lifetime ($J \to \infty$) (b) Finite lifetime (here $J = 4$)

Figure 3: Optimal choices for consumption c, housing stock h and assets a.

Lemma 1 and Proposition 1 show that optimal choices are constant over time and proportional to discounted lifetime income. An agent buys her optimal house in the first period, irrespective of how low the initial endowment is. To cover the gap between initial resources and the downpayment, the agent takes out a mortgage m_0 and repays it over time. Let m_t be the level of outstanding mortgages at the beginning of period t and π_t the debt service in period t. The law of motion is given by

$$m_t = (1+r)(m_{t-1} - \pi_t). \tag{1}$$

Proposition 2. *For homeowners, initial outstanding mortgages are*

$$m_0 = (\theta - 1)\left(\frac{y}{1+\theta\frac{\kappa_3+\delta}{1-\delta}} - \frac{a_0}{\theta + \frac{1-\delta}{\kappa_3+\delta}}\right), \tag{2}$$

The debt service payment is constant overtime,

$$\pi_t = \pi = y - c - \delta ph,$$

and the beginning-of-period outstanding mortgage at age j is

$$m_t = \sum_{i=0}^{J-t-1}\left(\frac{1}{1+r}\right)^i (y - c - \delta ph). \tag{3}$$

Proof. See Appendix A.2. □

From Proposition 2 it follows immediately that mortgages are positive, as long as the income is sufficiently high or initial assets are sufficiently low.

Corollary 1. *Initial mortgages are positive, $m_0 > 0$, iff*

$$\frac{y}{a_0} > \frac{\delta + \kappa_3}{(1-\delta)}.$$

Proof. Follows immediately from the intermediate equation (10) in the proof of Proposition 2. □

If an agent inherits a sufficiently large initial endowment, she can finance the downpayment of the house, without the need for a mortgage. If the initial endowment exceeds the downpayment, the agent will be a saver. If, on the other hand, an agent does not inherit any initial endowment, the initial income will not be sufficient to cover the downpayment. She will need to take shift part of her lifetime income to the present using a mortgage.

Moreover, we can see the determinants of indebtedness.

Corollary 2. *For homeowners, initial debt is increasing in the taste for housing ξ and flow income y and decreasing in initial endowments a_0. Outstanding debt is decreasing with age.*

Proof. Follows immediately from (2) and (3) in Proposition 2 because $\kappa_3 \propto 1/\xi - 1$ and $\frac{\partial \kappa_3}{\partial \xi} < 0$. □

There are three reasons, why households are more indebted than others: being young, having a stronger taste for housing, having low initial endowments relative to lifetime income. Households that are younger are more indebted, because they have had less time to repay their mortgage. Households that have a stronger taste for housing are more indebted because they need to finance a bigger house. Finally, for a given lifetime income, households with low initial endowments earn a larger share of their incomes later in life. That is, they need to shift a larger amount of their lifetime income to the present to finance the downpayment of the house.

2.2 Renters

Renters have no bequest motive. Their problem is then given by

$$\begin{aligned}
&\max_{\{c_t,h_t\}_{t=0}^{J-1}} \sum_{t=0}^{J-1} \beta^t u(c_t^{1-\xi} h_t^{\xi}) \\
&\text{s.t. } c_t + \rho h_t + a_{t+1} = (1+r)a_t + y_t \\
&\quad a_J \geq 0
\end{aligned}$$

where ρ denotes the price of renting one unit of the housing good.

Under our given assumptions, agents' consumption choice will not depend on the rental price (which is a function of the house price).

Proposition 3. *Optimal policies of renters are constant across time. Furthermore, the level of consumption is independent of the cost of renting,*

$$c^* = (1-\xi)\frac{\mathcal{Y}}{\theta}, \qquad \rho h^* = \xi \frac{\mathcal{Y}}{\theta}.$$

Proof. See Appendix A.3. □

In this framework renters' optimal consumption is independent of the cost of rent. Now suppose, that rent increases with rising house prices and vice versa. A decrease in house prices then, which reduces consumption of home owners, has no effect on renters. Their wealth is unaffected and therefore also spending on consumption. This is in line with e.g. Berger et al. (2018); Aladangady (2017) who find very small reactions of renters' consumption expenditure to changes in house prices.

3 Housing wealth effects with closed forms

We can now derive the main result of this paper: closed forms for the consumption response to house price shocks. We have already shown that there are no housing wealth effects for renters, so the remaining work to do is to derive results for homeowners.

We assume that house price shocks are unexpected and permanent. In our thought experiment, an agent wakes up at age j and observes that the house price has fallen from p to q. She reconsiders her optimal choices given her net worth,

$$\tilde{a}_j = q(1-\delta)h - (1+r)m_{j-1},$$

her unchanged flow income y and her remaining lifetime $J-j$. Due to exponential discounting, her optimal choices are time consistent (Strotz, 1955) and will be as if she was a $(J-j)$-period-lived agent with initial endowment $\tilde{a}_j$ and flow income y given house price q.

Proposition 4. *After an unexpected price change from p to q at the beginning of a period, a homeowner of age j will adjust their consumption*

$$\frac{c_j^*}{c_0^*} = \frac{(1-\delta)\frac{q}{p} + \theta^{J-j}(\delta+\Omega)}{(1-\delta) + \theta^{J-j}(\delta+\Omega)}$$

Proof. See Appendix A.4. □

Given this closed form result, it is easy to analyze the heterogeneity in housing wealth effects along different household characteristics.

Proposition 5. *The consumption response to an unexpected negative house price shock is (i) zero for renters and (ii) negative for homeowners. Moreover, the absolute response for homeowners is*

1. *increasing in age j and*
2. *increasing in the utility weight for housing ξ.*

Proof. See Appendix A.5. □

Homeowners that are older or have stronger preferences for housing are hit harder by house price shocks. This is illustrated in Figure 4. Agents have to reduce their consumption to compensate their losses in housing wealth. Intuitively,

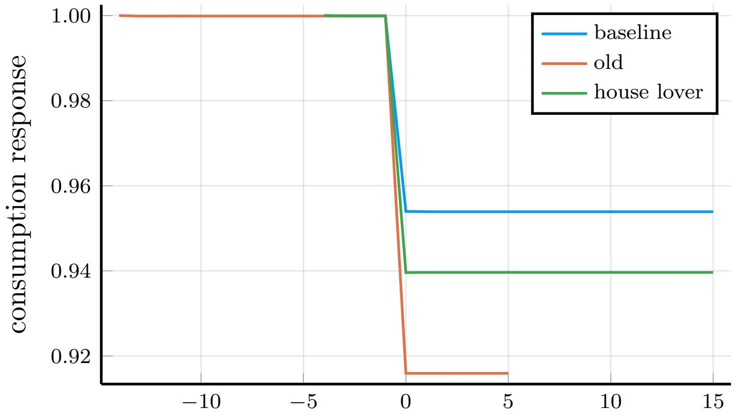


Figure 4: Housing wealth effects—the consumption response to a drop in house prices—by age and housing preferences. Shock happens in $t = 0$.

older agents react more strongly, because they have less time to smooth out their losses. Agents with stronger preferences for housing own a larger house, so they are facing larger losses that have to be compensated.

Housing wealth effects and indebtedness

Proposition 5 is silent about the role of debt on housing wealth effects. This is because indebtedness endogenous, rather than a primitive of the model. Rather than looking at the role of debt directly, we can analyze how the *drivers* of debt (see Corollary 2) affect the strength of housing wealth effects. Each driver—income profile, age and taste for housing—acts on housing wealth effects differently.

Income profile (y vs a_0) Varying the ratio of initial endowment and flow incomes will change the debt holdings for a given lifetime income $\mathcal{Y}$. If more of the lifetime income is earned through flow income, optimal debt will be higher. For the choices of c and h however, the composition of lifetime income is irrelevant in our complete markets setup. So, more indebted agents react *equally strongly.*

Age j Households repay their debt over their lifetime. Older agents are less indebted than poor agents. As shown above, older agents have a stronger consumption response. When comparing agents of different ages, more indebted agents react *less strongly.*

Taste for housing ξ We have shown that agents with stronger preferences for houses, are more indebted. They also react stronger to house price changes. When comparing agents of different housing preferences, more indebted agents react *more strongly.*

From the perspective of our model, the effect of debt on individual housing wealth effects is ambiguous. The reason for being indebted determines the strength of the consumption response to a change in house prices.

3.1 Rationalizing the findings in the empirical literature

Campbell and Cocco (2007) use survey data from the UK to find heterogeneous housing wealth effects with respect to housing status and age. They find the strongest reactions for older homeowners and the weakest reaction for young renters. To do so they look at changes in house prices across three regions (North, Center, South).[4] Due to data limitations, Campbell and Cocco (2007) cannot condition their findings on individual house size. Our model is consistent with their findings: older people react more strongly, renters do not react.

Aladangady (2017) links the individual expenditure data from the CEX with house price information on the MSA-level, using restricted-use geographical information from the CEX. He shows that the response to house prices is neglibile for renters and large for homeowners. His specifications implies that the consumption response is proportional to initial house values. Additionally, he finds that households with low LTV ratio react more strongly than households with high LTV ratio. Our model is consistent with the finding that effects are stronger for homeowners with bigger houses. On the other hand, according to our model, his estimated effect of the LTV ratio must pick up the underlying effect of the taste for housing.

Mian et al. (2013) use aggregate data (county and ZIP code level) on expenditures and household balance sheets to show that there the elasticity of consumption out of housing wealth is higher more leveraged and poorer households. While we cannot (yet) make a statement about the reaction of poorer households, we can rationalize the stronger effects of more indebted regions. If the age distribution is similar across regions, their estimate is simply picking up the effect of differences in the taste for housing. Regions with a stronger taste for housing will react more strongly according to our model.

3.2 Aggregate housing wealth effects

From Corollary 4 and Proposition 3 we know the housing wealth effects for owners and renters. We showed that under Cobb-Douglas aggregation, renters do not react at all.

Thus we can write the aggregate response as

$$\text{Homeownership rate} \times \text{mean response of owners.}$$

[4] This coarse distinction masks a lot of heterogeneity within these regions. For example, the region "North" contains all of Scotland with more densely populated, urban areas around Edinburgh or Glasgow and very sparsely populated in the north of Scotland. Hence, people may very well be assigned to a housing wealth effect when, in reality, they do not experience one (e.g. when there is strong price appreciation in London, affecting the whole region). We are able to avoid this problem by only considering narrowly defined MSAs.

Thus, bigger homeownership rate will lead to a stronger aggregate consumption response to a house price change.

4 Testing the predictions on housing wealth effects

Now we take the model predictions to the data. We combine the CEX, a quarterly panel with MSA-level geographic identifiers, with regional house price data from Zillow.com. We construct a simple measure of housing preferences from the residual of a regression explaining house sizes. We show that, in line with our model, age and housing preferences are significant predictors of the size of housing wealth effects. When these two explanatory variables are ommitted, their effect is picked up by the level of mortgage debt.

4.1 Data

For the empirical exercise we employ data from two sources. First, we obtain publicly available house price indices on the MSA level from Zillow.com, a real estate listing site. In particular, we use the Zillow Home Value Index, which is a "smoothed, seasonally adjusted measure of the median estimated home value across a given region and housing type" according to the website.[5] This data has been used in several other papers such as Graham (2018). The data set covers the period from 1996 until 2017 on a monthly frequency, which includes both the sharp decline following the financial crisis as well the strong recovery in house prices that followed. This is an advantage over many other papers in the literature that often only look at the sharp increase in prices prior to the crisis.

Secondly, we use publicly available data from the Consumer Expenditure Survey (CEX) from the Bureau of Labor Statistics. The CEX is a household survey which includes detailed information on expenditures (such as durable and non-durable goods). Additionally, the survey contains information on the housing status of the household (i.e., if the household is a renter, homeowner etc.), mortgage information and some other, more general household characteristics. Households are observed at most four times within in 12 months (the time period does not have to correspond to a calendar year). Between observations, there are always three months. This structure gives the data set a panel dimension which allows us to identify the effect across time.[6]

To match both data sets we use the fact that the publicly available CEX data includes geographical information on the MSA level for a subset of all households (23 MSAs). Hence, we allocate the house price level (measured by the index) in a given month to every household which was residing in that particular MSA. Our final data set consists of around 37.000 unique households observed between 2006 and 2017.

[5] More information and the detailed methodology can be found under https://www.zillow.com/research/data/

[6] For more details on the CEX data see https://www.bls.gov/cex/pumd.htm.

4.2 Constructing a measure of housing preferences

The model predicts that the utility weight of housing ξ is an important determinant of the magnitude of housing wealth effects. Since preferences for housing are not directly observable, we need to construct a proxy from the available data. We use information on *rent equivalents*, which is the imputed market rent for a household's house or apartment. The basic idea behind this approach is that, given a household's observable characteristics (such as income and size), a higher implicit price one would pay for housing suggests stronger preferences for housing (consistent with our model). To operationalize this idea, we run a regression explaining the rent equivalents using a set of household characteristics $\boldsymbol{x}_i$ (such as location, education, age, family size and income),

$$\text{rent}_i = \alpha + \boldsymbol{\beta x}_i + u_i.$$

Then we define the measure of housing preferences as the residual of the regression (as a percentage deviation),

$$\text{pref}_i = \frac{\text{rent}_i - \widehat{\text{rent}}_i}{\widehat{\text{rent}}_i}.$$

A household with a larger residual has a larger (or more expensive) home than households with similar characteristics. We interpret that as the household having larger than average utility weight of housing ξ.

4.3 Results

We run a regression of the form

$$\frac{\Delta c_{i,a,t}}{\Delta p_{a,t}} = \boldsymbol{x}_{i,a,t}\boldsymbol{\gamma} + \varepsilon_{i,a,t},$$

where a is the index for the MSA, $\Delta c_{i,a,t}$ is household-level change in non-durable consumption and $\Delta p_{a,t}$ in the change in the MSA-level house price index and $\boldsymbol{x}_{i,a,t}$ contains the a subset variables of interest (mortgages, age, the preference proxy, home ownership) and MSA and year fixed effects as control variables.

We find that this simple test supports the predictions of our model. The log of outstanding mortgages is a significant predictor of the size of housing wealth effects—but only as long as the determinants of mortgages are excluded from the regression. Indeed, regression (2) in Table 1 shows that the housing preference measure and age are statistically significant predictors of housing wealth effects, when mortgages are excluded. When including all three variables in regression (3) of Table 1, they all turn insignificant. This result reflects the fact that there is a high correlation between these variables.

Table 1: Regression output

	$\Delta c/\Delta p$		
	(1)	(2)	(3)
own × log(mortgage)	0.727**		0.746°°
	(0.362)		(0.523)
own × h-pref-proxy		2.444**	1.722°
		(1.229)	(1.371)
own × age		0.323*	0.247
		(0.177)	(0.269)
own × age^2		-0.003*	-0.002
		(0.002)	(0.003)
MSA FE	Yes	Yes	Yes
year FE	Yes	Yes	Yes
Estimator	OLS	OLS	OLS
N	36,350	52,307	33,796
R^2	0.002	0.001	0.002

$^{***}p \leq 0.01$, $^{**}p \leq 0.05$, $^{*}p \leq 0.1$, $^{\circ\circ}p \leq 0.2$, $^{\circ}p \leq 0.3$

5 Conclusion

Empirical and quantitative macroeconomic studies have found that housing wealth effects are stronger for more indebted households. One important policy implication is that lowering debt limits for borrowers will dampen the consumption slump in a house price bust. In this chapter we show that such conclusions might be premature.

We build a simple life-cycle model with housing with closed form solutions for housing wealth effects. We show that the strength of housing wealth effects crucially depends on the underlying household characteristics which also determine the debt levels. In this framework imposing one-size-fits-all debt limits does not necessarily mitigate housing wealth effects. To be effective, policies have to be tailored to borrowers' characteristics. Aggregate housing wealth effects can be reduced in three ways: (i) if old homeowners reduce their housing wealth; (ii) if the home ownership rate decreases; (iii) if agents have smaller houses. We provide a simple empirical test of our model predictions. When explaining housing wealth effects, we find that the level of mortgages turns statistically insignificant once relevant household characteristics (age and a proxy for housing preferences) are added.

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A Proofs

A.1 Proof of Lemma 1

Proof. The Lagrangian is given by

$$\sum_{t=0}^{J-1} \beta^t \left(u(c_t, h_t) - \lambda_t \Big(a_{t+1} - (1+r)a_t - y_t + c_t + p(h_t - (1-\delta)h_{t-1}) \Big) \right) + \beta^{J-1}\psi(h_{J-1})$$

where $a_J = 0$ is given. The first order conditions are as follows. For a_t, $t \leq J-1$,

$$\lambda_{t-1} = \beta(1+r)\lambda_t \implies \lambda_0 = \cdots = \lambda_{J-1} = \lambda.$$

For c_t for $t \leq J-1$,

$$u_c(c_t, h_t) = \lambda_t = \lambda$$

For h_t for $t < J-1$,

$$\begin{aligned} u_h(c_t, h_t) &= \lambda_t p - (1-\delta)p\beta\lambda_{t+1} = \lambda p(1-\beta(1-\delta)) \\ \implies \frac{u_h}{u_c} &= p(1-\beta(1-\delta)) \end{aligned} \tag{4}$$

and for $t = J-1$

$$u_h(c_{J-1}, h_{J-1}) = \lambda_{J-1} p - \psi_h(h_{J-1}). \tag{5}$$

Using the CRRA-Cobb-Douglas functional form assumption we get

$$u_c(c,h) = (1-\xi)\frac{(c^{1-\xi}h^{\xi})^{1-\gamma}}{c} \tag{6}$$

$$u_h(c,h) = \xi\frac{(c^{1-\xi}h^{\xi})^{1-\gamma}}{h} \tag{7}$$

$$\frac{u_h}{u_c} = \frac{\xi}{1-\xi}\frac{c}{h}. \tag{8}$$

Combining (4) and (8) yields

$$\frac{\xi}{1-\xi}\frac{c}{h} = p(1-\beta(1-\delta))$$

which gives an optimal relationship of c and h,

$$c^*(h) = \underbrace{(1-\beta(1-\delta))\frac{1-\xi}{\xi}}_{\kappa_3} ph = \kappa_3 ph. \tag{9}$$

Using this relationship, (6) and (7) simplify to

$$u_c(c^*(h),h) = (1-\xi)\frac{((\kappa_3 ph)^{1-\xi}h^{\xi})^{1-\gamma}}{\kappa_3 ph} = (1-\xi)h^{-\gamma}(\kappa_3 p)^{(1-\xi)(1-\gamma)-1}$$

$$u_h(c^*(h),h) = \xi\frac{((\kappa_3 ph)^{1-\xi}h^{\xi})^{1-\gamma}}{h} = \xi h^{-\gamma}(\kappa_3 p)^{(1-\xi)(1-\gamma)}.$$

We choose ψ to ensure that (9) also holds at age $J-1$. So we plug the previous expressions into (5),

$$u_h - p\lambda = u_h - pu_c = h^{-\gamma}(\kappa_3 p)^{(1-\xi)(1-\gamma)}\Big(\xi - p\frac{1-\xi}{\kappa_3 p}\Big) = \psi'(h).$$

Finally, define

$$\tilde{\kappa}_1 := \xi - p\frac{1-\xi}{\kappa_3 p} = \xi\frac{\beta(1-\delta)}{1-\beta(1-\delta)},$$

$$\tilde{\kappa}_2 := (\kappa_3 p)^{1-\xi},$$

and use the guess for ψ from Assumption 2 to determine the undetermined coefficients,

$$\psi'(h) = \kappa_1\kappa_2^{1-\gamma}h^{-\gamma} = h^{-\gamma}\tilde{\kappa}_2^{1-\gamma}\tilde{\kappa}_1.$$

$$\implies \kappa_1 = \tilde{\kappa}_1, \quad \kappa_2 = \tilde{\kappa}_2. \qquad \square$$

A.2 Proof of Proposition 2

From Proposition 1 and the flow budget constraint XX we know that the initial mortgage is

$$m_0 = ph + c - y - a_0.$$

Plugging in optimal choices we get

$$\begin{aligned}
&= (1+\kappa_3)\frac{\theta y + a_0}{1-\delta+(\delta+\kappa_3)\theta} - (y+a_0) \\
&= \frac{(1+\kappa_3)(\theta y + a_0) - (y+a_0)(1-\delta+(\delta+\kappa_3)\theta)}{1-\delta+(\delta+\kappa_3)\theta} \\
&= y\frac{(1+\kappa_3)\theta - (1-\delta+(\delta+\kappa_3)\theta)}{1-\delta+(\delta+\kappa_3)\theta} + a_0\frac{(1+\kappa_3) - (1-\delta+(\delta+\kappa_3)\theta)}{1-\delta+(\delta+\kappa_3)\theta} \\
&= y\frac{(\theta-1)(1-\delta)}{1-\delta+(\delta+\kappa_3)\theta} - a_0\frac{(\theta-1)(\kappa_3+\delta)}{1-\delta+(\delta+\kappa_3)\theta} \\
&= (\theta-1)\frac{y(1-\delta) - a_0(\kappa_3+\delta)}{1-\delta+(\delta+\kappa_3)\theta} \qquad (10) \\
&= (\theta-1)\left(\frac{y}{1+\theta\frac{\kappa_3+\delta}{1-\delta}} - \frac{a_0}{\theta+\frac{1-\delta}{\kappa_3+\delta}}\right),
\end{aligned}$$

which is the first claim of the proposition. The second claim follows from the flow budget constraint and the fact that y, h and c are constant over time (Proposition 1).

For the third claim, use the fact that $m_J = 0$, and solve the difference equation (1) forward,

$$\begin{aligned}
m_t &= \frac{1}{1+r}m_{t+1} + \pi_t \\
&= \frac{1}{1+r}\left(\frac{1}{1+r}m_{t+2} + \pi_{t+1}\right) + \pi_t \\
&= \left(\frac{1}{1+r}\right)^s m_{t+s} + \sum_{i=1}^{s}\left(\frac{1}{1+r}\right)^{i-1}\pi_{t+s-1}
\end{aligned}$$

let $J = t + s$

$$\begin{aligned}
&= \left(\frac{1}{1+r}\right)^{J-t}\underbrace{m_J}_{0} + \sum_{i=1}^{J-t}\left(\frac{1}{1+r}\right)^{i-1}\pi \\
&= \sum_{i=1}^{J-t}\left(\frac{1}{1+r}\right)^{i-1}\pi_t. \qquad \square
\end{aligned}$$

A.3 Proof of Proposition 3

As before the budget constraints can be combined into one lifetime budget constraint. Let $\mathcal{Y}$ denote lifetime income. Let λ denote the Lagrange multiplier of the constraint maximization problem. The FOCs of the new problem are given by

$$
\begin{aligned}
(1-\xi)u'(c_t^{1-\xi}h_t^{\xi})\Big(\frac{h_t}{c_t}\Big)^{\xi} &= \lambda \\
\xi u'(c_t^{1-\xi}h_t^{\xi})\Big(\frac{c_t}{h_t}\Big)^{(1-\xi)} &= \lambda\rho
\end{aligned}
$$

From here it follows that policies are constant over time. Furthermore, rearranging the FOCs and plugging them back into the lifetime budget constraint yields the optimal policies:

$$
\begin{aligned}
c^* &= (1-\xi)\frac{\mathcal{Y}}{\theta} \\
\rho h^* &= \xi\frac{\mathcal{Y}}{\theta}
\end{aligned}
$$

where θ is as defined before. The desired result follows.

A.4 Proof of Proposition 4

$$
\begin{aligned}
c^*(\mathcal{Y}_0, J) &= c^*(\mathcal{Y}_j, J-j) \\
h^*(\mathcal{Y}_0, J) &= h^*(\mathcal{Y}_j, J-j)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{Y}_0 &= \tilde{a}_0 + \theta^J y \\
\mathcal{Y}_j &= \tilde{a}_j + \theta^{J-j} y
\end{aligned}
$$

and

$$
\tilde{a}_j = (1-\delta)ph_{j-1} - m_j.
$$

If, however, the environment changes, the agent will want to reallocate their expenditures.

After the price change, agent's optimal choices are given by

$$
\begin{aligned}
h_j^* &= \mathcal{Y}_j \frac{1}{q\big((1-\delta)+\theta^{J-j}(\delta+\Omega)\big)} \\
c_j^* &= q\Omega h_j^*
\end{aligned}
\tag{11}
$$

where the new lifetime income at age j is given by a combination of current wealth (h and m) and future income

$$
\mathcal{Y}_j = \tilde{a}_j + \theta^{J-j} y
$$

with

$$\begin{aligned}\tilde{a}_j &= (1-\delta)qh - m_j \\ &= (1-\delta)qh - m_{J-(J-j))}.\end{aligned}$$

Plugging in our formulas for m_{J-t} (lemma XXX) and c (assumption XXX) we get

$$\begin{aligned}\tilde{a}_j &= (1-\delta)qh - \theta^{J-j}\pi \\ &= (1-\delta)qh - \theta^{J-j}(y - c - \delta ph) \\ &= \big((1-\delta)q + p\theta^{J-j}(\Omega+\delta)\big)h - \theta^{J-j}y.\end{aligned}$$

Thus, agents lifetime income at age j is

$$\mathcal{Y}_j = \big((1-\delta)q + p\theta^{J-j}(\Omega+\delta)\big)h_0^* \tag{12}$$

Combining equations (11) and (12) we get new optimal house at age j,

$$h_j^* = \frac{p}{q}\frac{(1-\delta)\frac{q}{p} + \theta^{J-j}(\delta+\Omega)}{(1-\delta)+\theta^{J-j}(\delta+\Omega)}h_0^*$$

The new optimal consumption level at age j is given by

$$c_j^* = q\Omega h_j^* = \frac{p}{q}\frac{(1-\delta)\frac{q}{p} + \theta^{J-j}(\delta+\Omega)}{(1-\delta)+\theta^{J-j}(\delta+\Omega)}\frac{q}{p}\underbrace{p\Omega h_0^*}_{c_0^*}$$

The optimal consumption response to an unexpected house price shock follow directly from the previous equation.

A.5 Proof of Proposition 5

The consumption response has the following structure,

$$\frac{c_j^*}{c_0^*} = \frac{a+f(x)}{b+f(x)}$$

where $a = (1-\delta)\frac{q}{p}$, $b = (1-\delta)$ and $f(J, j, \delta, \Omega) = \theta^{J-j}(\delta+\Omega)$. The derivative is given by

$$\begin{aligned}\frac{\partial}{\partial x}\frac{c_j^*}{c_0^*} &= \frac{f'(x)(b+f(x)) - f'(x)(a+f(x))}{\big(b+f(x)\big)^2} \\ &= \frac{f'(x)(b-a)}{\big(b+f(x)\big)^2} \\ &= f'(x)\frac{(1-\delta)(p-q)}{p\big(1-\delta+f(x)\big)^2} \\ &\propto f'(x)\end{aligned}$$

for a ngetive shock to prices. That is we can look at the partial derivative of f only.

$$\frac{\partial f}{\partial \Omega} = \theta^{J-j} > 0$$

Higher Ω means a bigger weight on consumpion, that is agents *hate houses more*. Thus,

$$0 < CR(\Omega_L) < CR(\Omega_H) < 1,$$

or

$$-100\% < \%CR(\Omega_L) < \%CR(\Omega_H) < 0\%.$$

That is, agents who love house more (lower Ψ), react stronger. For the following result consider the extension of θ to the real numbers,

$$\theta(t,r) = \frac{1 - \left(\frac{1}{1+r}\right)^t}{1 - \frac{1}{1+r}}$$
$$\frac{\partial f}{\partial j} = (\delta + \Omega)\underbrace{\theta'(J-j,r)}_{>0}(-1) < 0$$

That is, agents with higher age react stronger (Using the same reasoning as above).