Falling Behind: Has Rising Inequality Fueled the American Debt Boom?*

Moritz Drechsel-Grau Fabian Greimel

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Abstract

We investigate whether rising income inequality and *Keeping up with the richer Joneses* (KURJ) fueled the US mortgage and housing boom. This is motivated by two novel empirical findings. First, both household debt of the non-rich and house prices grew substantially more in US states where top incomes grew faster. Second, higher top incomes drive up mortgage debt but do not affect *non*-mortgage debt. These stylized facts cannot be generated by previously studied drivers of the debt boom. Thus, we propose rising income inequality and KURJ as a complementary causal driver of the debt boom. To that end, we build a heterogeneous agent macroeconomic model where households care about how their house compares to the benchmark set by the rich. We show analytically that mortgage debt of the non-rich is increasing in top incomes. This is because the non-rich substitute status-enhancing housing for status-neutral consumption to keep up with the upgraded houses of the rich. This mechanism is quantitatively important, generating up to 60% of the observed increase in mortgage debt and up to 50% of the observed increase in house prices. In comparison, we find that the Global Saving Glut gives rise to a similar debt boom, but does not generate a house price boom.

Keywords: mortgages, housing boom, social comparisons, consumption networks, keeping up with the Joneses

JEL Codes: D14, D31, E21, E44, E70, R21

*Drechsel-Grau: University of Zurich; moritz.drechsel-grau@uzh.ch and www.moritzdrechselgrau.com. Greimel: University of Amsterdam; f.greimel@uva.nl and www.greimel.eu. We are indebted to our advisors Klaus Adam, Tom Krebs, Andreas Peichl, Sebastian Siegloch and Michèle Tertilt for their guidance and support. This paper has benefitted from formal conference discussions by Alina Bartscher, Leo Kaas, Christian Kellner and Dayin Zhang as well as feedback by Andrey Alexandrov, Antoine Camous, John Geanakoplos, Dan Greenwald, Hans Peter Grüner, Andreas Gulyas, Anne Hannusch, Lukas Henkel, Benjamin Larin, Matthias Meier, Ben Moll, Kurt Mitman, Galo Nuño, Krzysztof Pytka, Timo Reinelt, Alireza Tahbaz-Salehi, Tony Smith and Amir Sufi, all at very different stages of this project. We thank members of the audience at the virtual WFA Meetings 2020, the SED Meetings 2018 in Mexico City, the ESWM 2019 in Rotterdam, the ESSFM 2018 in Gerzensee, CEPR Conferences in Mannheim and Frankfurt, the Frankfurt–Mannheim Macro Workshop 2019, the ESEM 2018 in Cologne, the VfS Meeting 2018 in Freiburg for their input. Drechsel-Grau gratefully acknowledges funding by the German Research Foundation (DFG) through CRC TR 224 (Project A04). Greimel gratefully acknowledges funding by the foundation *Geld & Währung* and the German Research Foundation (DFG) through CRC TR 224 (Project C02) and SFB 884.

1 Introduction

Between 1980 and 2007, US household debt doubled relative to GDP. Mortgage debt was by far the most important driver of this household debt boom (see Figure 1a). In lockstep with mortgages, income inequality started to rise in 1980 and reached its peak in 2007 (see Figures 1b and 2). While real incomes stagnated for the bottom half of the population, incomes of the top 10% more than doubled over this time period (see Figure 2). The rise in household debt has drawn a lot of interest that has mostly focused on the role of falling interest rates following an increase in foreign and domestic *supply of credit* (e.g. Bernanke, 2005; Mian, Straub, and Sufi, 2021). In this paper, we investigate whether rising income inequality and *Keeping up with the richer Joneses* fueled the mortgage boom through an increase in the *demand for housing*.

We begin by documenting novel aspects of the US mortgage and housing boom which call for such a demand-side mechanism to complement supply-side drivers of household debt. First, we document that the aggregate link between top incomes and household debt extends to the level of US states. That is, household debt grew substantially more in US states where top incomes grew faster. While falling interest rates clearly fueled the rise in household debt, they are not able to account for this state-level link between top incomes and mortgage debt because financial markets are largely integrated and local demand for savings need not be compensated by local debt. Instead, arbitrage should lead to a uniform increase in debt of the non-rich across all states. Second, we show that higher state-level top incomes drive up mortgage debt but do not affect state-level non-mortgage debt. This suggests that housing plays a key role in the transmission from rising top incomes to rising household debt. Third, we find that house prices grew faster in states with a stronger increase in top incomes pointing to an important role for housing demand. Most importantly, we show that an increase in the average incomes of the rich is associated with an increase in the mortgage-to-income ratio of the non-rich—a key prediction of of the *Keeping up with the richer Joneses* (KURJ) mechanism.

In the main part of the paper, we assess—analytically and quantitatively—the aggregate consequences of rising income inequality in the presence of social comparisons. The idea that people compare themselves to others was introduced into economics by Veblen (1899) and Duesenberry (1949) and is supported by plenty of recent empirical research.[1] We incorporate social comparisons into a heterogeneous agent model of the macro economy. In our model, households not only care about their own consumption and housing, but also about how their house compares to the benchmark set by the rich. When top incomes rise and the rich upgrade their houses, the non-rich lose some of their social status and substitute status-enhancing housing for status-neutral consumption to keep up with the richer Joneses. These houses are mortgage-financed, causing a boom in debt-to-income ratios across the entire income distribution as well as an increase in house prices.

[1] See for example Luttmer (2005), Charles, Hurst, and Roussanov (2009), Kuhn, Kooreman, Soetevent, and Kapteyn (2011), Bursztyn, Ederer, Ferman, and Yuchtman (2014), or De Giorgi, Frederiksen, and Pistaferri (2020). Importantly, there is recent micro evidence by Bellet (2019) showing that (i) comparisons are upward-looking such that only the houses of the rich positional externality.

In a stylized version without idiosyncratic income risk, we can show analytically how this status externality affects aggregate debt depending on who cares about whom in the network of social comparisons. Allowing for arbitrary connections among income types, we prove that a household's housing demand and her debt level is increasing in the incomes of her reference group (or the reference groups of her reference group). This is because these incomes determine the reference measure of housing. In the empirically relevant case where non-rich households care about the rich, the debt-to-income ratio of the non-rich is increasing in the incomes of the rich.[2] This implies that aggregate debt-to-income is increasing in top incomes.

We then calibrate the full model with idiosyncratic income risk in order to quantify the contribution of this mechanism to the observed increase in mortgages and house prices between 1980 and 2007. We discipline the social comparison motive using independent micro evidence on housing comparisons in the US by Bellet (2019) who estimates how strongly households' utility is affected by the housing of the local rich. The main experiment is to compare two steady states that differ only in the exogenous degree of income inequality. In particular, we scale the *permanent* component of income inequality to match the increase in cross-sectional income dispersion between 1980 and 2007.[3]

We find that in the presence of KURJ, the rise in income inequality can explain up to half of the observed 120%-increase in the mortgage-to-income ratio and up two thirds of the observed 60%-increase in house prices between 1980 and 2007. This effect can be decomposed into a direct effect and an indirect effect. On the one hand, social comparisons directly raise housing demand and thereby mortgage demand for non-rich households. On the other hand, rising inequality drives up house prices through growing demand for housing at the top of the income distribution. As housing and non-durable consumption complement each other, this increase in the equilibrium house price pushes up housing and mortgage demand of non-rich households. Even in the absence of KURJ, rising inequality can explain about a quarter of the observed mortgage boom through this general equilibrium effect. The model also accounts for up to half of the observed 65%-increase in the house-value-to-income ratio between 1980 and 2007.

Finally, we compare the effects of our demand-side mechanism to those of the *Global Saving Glut*, i.e. the surge in the foreign net debt position of the US from about 0% of GDP in 1980 to about 40% of GDP in 2007 (Bernanke, 2005; Justiniano, Primiceri, and Tambalotti, 2014).[4] In our model, this increase in the supply of credit can account for about 30% of the debt boom through lowering the real interest rate by approximately 40%. In contrast to rising inequality and KURJ, however, we find that the Global Saving Glut increases house prices by only 2% and the ratio of house values to income by only 4%.

[2] Bellet (2019) shows that households only care about the top end of the housing distribution. To our knowledge all papers that have tested for asymmetries in the comparison motive have found them to be upward-looking (e.g. Clark and Senik, 2010; Ferrer-i-Carbonell, 2005; Card, Mas, Moretti, and Saez, 2012).

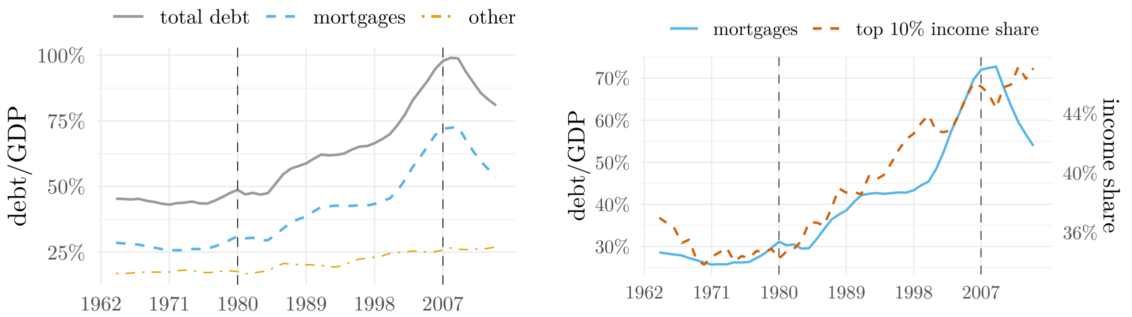
[3] This focus on permanent income inequality is in line with evidence in Kopczuk, Saez, and Song (2010) and Guvenen, Kaplan, Song, and Weidner (2018) who show that the rise in cross-sectional inequality is mostly driven by permanent income differences rather than increasing risk.

[4] Gourinchas, Rey, and Govillot (2017) show that the net foreign debt position can be well approximated by the cumulative current account deficit.

Figure 1: The American Household Debt Boom and Rising Income Inequality

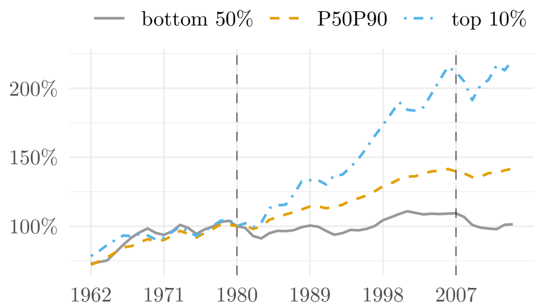
(a) Decomposition of the Debt Boom

(b) Mortgages (left axis) and Inequality (right)



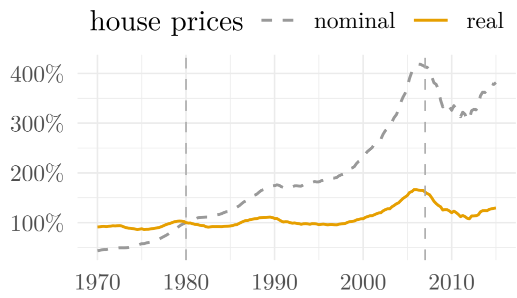
Notes: This figure shows the relationship of aggregate household debt (total, mortgage, non-mortgage) as share of GDP and the top 10% income share over time. Data sources: US Flow of Funds and Alvaredo, Atkinson, Piketty, Saez, and Zucman (2016). Details see Appendix B.

Figure 2: Distribution of Income Growth



Notes: This figure shows real average pre-tax income growth from 1962 to 2014 in the US. Data are taken from Piketty, Saez, and Zucman (2018). Growth rates are relative to the base year 1980.

Figure 3: House Prices in the US



Notes: Nominal: Case-Shiller Home Price Index for the USA. Real: Deflated by the Consumer Price Index. Base year: 1980. Source: http://www.econ.yale.edu/~shiller/data.htm

Both mechanisms together can explain up to three quarters of the observed 120%-increase in the mortgage-to-income ratio. Decomposing this total effect, we can attribute between one half and two thirds of the explained increase in debt to rising inequality and KURJ.

Related Literature. Our paper relates to several strands of the literature. First, we contribute to the literature that studies the drivers of the US household debt boom which was documented by Jordà, Schularick, and Taylor (2016) and Kuhn, Schularick, and Steins (2017). A range of papers focuses on an increase in the foreign or domestic supply of credit that drives up household debt through a drop in the interest rate (Justiniano et al., 2014; Kumhof, Rancière, and Winant, 2015; Mian, Straub, and Sufi, 2020; Mian et al., 2021). Most notably, Mian et al. (2021) show that differences in saving rates out of permanent income can link rising income inequality to rising credit supply and falling interest rates. Other papers study the role of looser collateral constraints (e.g. Favilukis, Ludvigson, and van Nieuwerburgh, 2017) and lending limits (Justiniano, Primiceri, and Tambalotti, 2019) as well as changes in house price expectations (Adam, Kuang, and Marcet, 2012; Kaplan, Mitman, and Violante, 2020). This paper adds to this literature in two ways: First, we document a tight link between (non-rich) mortgage debt and top incomes on the state level. Second, we explore a novel demand-side mechanism that can help rationalize the tight link between top incomes and non-rich debt and complements the existing supply-side mechanisms.[5]

Second, we contribute to the growing literature on the aggregate effects of rising income inequality. Heathcote, Storesletten, and Violante (2010) analyze how rising wage inequality affects human capital investment and labor supply. Auclert and Rognlie (2018) investigate how permanent and transitory income inequality differentially impact aggregate demand. Straub (2018) shows that rising permanent income inequality drives down interest rates in the presence of non-homothetic preferences. Fogli and Guerrieri (2019) analyze the interplay between residential segregation and income inequality in the presence of local spillovers that affect the education returns. The approach of Straub (2018) and in particular Fogli and Guerrieri (2019) is similar to ours in the sense that we integrate insights from empirical microeconomic research into (non-homothetic preferences, local spillovers, social comparisons) into a macroeconomic heterogeneous agent model to analyze potential interactions with rising inequality. In our model, agents are linked not only through prices but also directly through social externalities of their consumption decisions.

Third, we contribute to the large literature on social comparisons (e.g. Luttmer, 2005; Card et al., 2012; Perez-Truglia, 2019) and economic choices (Charles et al., 2009; Kuhn et al., 2011; Bursztyn et al., 2014; Bertrand and Morse, 2016; Bursztyn, Ferman, Fiorin, Kanz, and Rao, 2017; Bellet, 2019; De Giorgi et al., 2020). While the macroeconomic effects of keeping up with the Joneses have already been studied in the context of representative agent models (e.g. Abel, 1990; Campbell and Cochrane, 1999; Ljungqvist and Uhlig, 2000), we introduce social comparisons into a quantitative heterogeneous agents model. We build on the macroeconomic literature on keeping up with the Joneses and bring it closer to the empirical evidence. First, we distinguish between conspicuous and non-conspicuous goods. In our model households compare themselves only in their houses, arguable the most important conspicuous good (e.g. Solnick and Hemenway, 2005; Bertrand and Morse, 2016). And second, agents compare themselves to the rich (e.g. Card et al., 2012; Bellet, 2019). Households only lose satisfaction with their own house, when a big house is built.[6]

Fourth, our empirical results are most closely related to the the studies by Bertrand and Morse (2016) who use CEX data and state-year variation to document that consumption expenditures of non-rich households respond to the incomes and consumption expenditures of the rich. Coibion, Gorodnichenko, Kudlyak, and Mondragon (2020) investigate the relationship between zip-code level income inequality (P90-P10 ratio) and household debt between 2000 and 2012 and find heterogeneous effects by income rank. Mian et al. (2020) analyze whether increasing top incomes in a state lead to an increase in the amount of non-rich household debt held as an asset by the state's rich. We analyze whether the

[5] Note that the question whether rising top income inequality fueled the boom in household debt and amplified the Great Recession was also discussed in the public debate (e.g. Rajan, 2010; Stiglitz, 2009; Frank, 2013). See also the survey by van Treeck (2014).

[6] These novel modeling choices distinguish our paper from Badarinza (2019), who shows that status externalities lead to inefficient debt levels in a lifecycle model.

state's non-rich take on more debt and analyze the dynamic effects of increases in top incomes. In addition, we show that growing top incomes are also associated with higher state-level house prices.

Finally, our analytical results extend those by Ghiglino and Goyal (2010) and Ballester, Calvó-Armengol, and Zenou (2006) who show that agents' choices depend on the strengths of social links in a one-period model. We extend their network models to infinite horizon and add a durable good (housing) to show that debt is increasing in the centrality of an agent. The centrality is reinterpreted as the weighted sum of incomes of the comparison group.

Structure of the paper The rest of the paper is structured as follows: In Section 2 we present empirical evidence on the relationship between household debt and top income inequality. In Section 3 we describe our model. In Section 4 we derive analytically how top incomes drive debt in a stylized version of the model. In Section 5 we describe the parameterization of the full model, followed by quantitative results in Section 6. Section 7 concludes.

2 Empirical Analysis: Top Incomes and Household Debt

In this Section, we use state-level distributional national accounts (DINA) data (Piketty et al., 2018; Mian et al., 2020) to study the relationship between US top incomes and household debt in more detail. We show that the aggregate link between top incomes and (non-rich) household debt goes beyond mere coincidence. Our empirical analysis exploits state-year variation in top incomes after controlling for aggregate shocks and time-invariant state heterogeneity. Let us emphasize at the outset that we do not use an explicit source of quasi-experimental variation in top incomes. Instead, we follow Mian et al. (2020) and argue that plenty of evidence in the literature supports the view that the rise in top inequality was triggered by shifts in technology and globalization that took place at the outset of the rise in inequality around 1980 (e.g. Katz and Murphy, 1992; Autor, Katz, and Kearney, 2008; Smith, Yagan, Zidar, and Zwick, 2019).

2.1 Data & Approach

We use state-level data on incomes and debt between 1980 and 2007 adapted from the data provided by Mian et al. (2020). These data are based on DINA data from Piketty et al. (2018). As state-level identifiers in the DINA data are suppressed for incomes above 200,000 US dollars, state identifiers are imputed using state-level data from the Internal Revenue Service (IRS) which include information on how many tax returns above 200,000 dollars come from each state.[7] Our main data set is a state-year panel for the period

[7]The imputation is based on the assumption that incomes above 200 thousand dollars follow a state-specific Pareto distribution with density $f_s(y) = \frac{\alpha_s 200{,}000^{\alpha_s}}{y^{\alpha_s+1}}$ where α_s can be computed from the state-level mean income of units with gross income above 200,000 dollars. The ratio of the state-specific and aggregate income density gives the relative likelihood that an observation comes from that state. This is then used to weight all observations when computing state averages.

Table 1: Top Incomes and Household Debt: Fixed Effects Regressions

	log(debt$_{s,t}$)					
	(A) total		(B) mortgage		(C) non-mortgage	
	population	non-rich	population	non-rich	population	non-rich
	(1)	(2)	(3)	(4)	(5)	(6)
log(top incomes$_{s,t-2}$)	0.108*	0.215***	0.180**	0.306***	-0.156***	-0.054
	(0.059)	(0.070)	(0.079)	(0.099)	(0.050)	(0.054)
log(own income$_{s,t}$)	0.678***	0.796***	0.764***	0.934***	0.463***	0.519***
	(0.093)	(0.060)	(0.119)	(0.087)	(0.051)	(0.057)
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	OLS	OLS	OLS
N	1,224	1,224	1,224	1,224	1,224	1,224
R^2	0.983	0.979	0.974	0.965	0.985	0.986

Notes: This table shows the estimation results corresponding to equation 1. The dependent variable is either total, mortgage or non-mortgage debt in the population or among the non-rich. Robust standard errors in parentheses. The stars indicate the range of the p value: $^{***} \leq 0.01 \leq {}^{**} \leq 0.05 \leq {}^{*} \leq 0.1$.

1980–2007 covering income, outstanding mortgages, outstanding non-mortgage debt and outstanding total debt for different income groups such as the rich (top 10%) and the non-rich (bottom 90%). We complement these data with state-level data on house prices from the Federal Housing Finance Agency and consumer prices from Hazell, Herreño, Nakamura, and Steinsson (2020).

2.2 Top Incomes and Household Debt: Fixed Effect Regressions

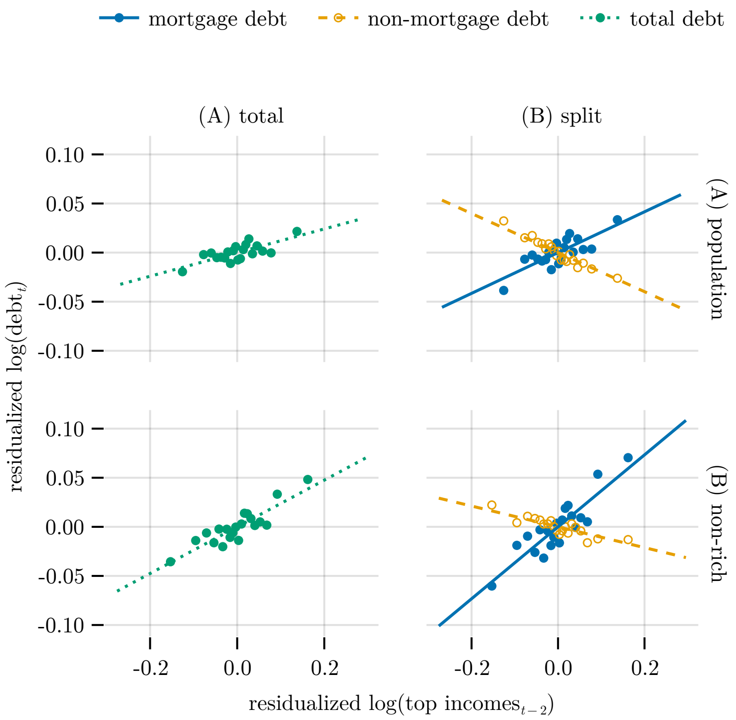
The main explanatory variable is the log of lagged top incomes measured as the average income in the top 10%.[8] Let debt$^{g}_{s,t}$ be either total, mortgage or non-mortgage debt in sub-population g in state s at time t. In our main estimation equation, we regress debt of group g on lagged top incomes, income of group g as well as state and year fixed effects.

$$\log(\text{debt}^{g}_{s,t}) = \beta \log(\text{top incomes}_{s,t-2}) + \gamma \log(\text{incomes}^{g}_{s,t}) + \delta_s + \delta_t + \epsilon_{s,t} \tag{1}$$

If β is positive, higher top income levels are associated with higher levels of future (non-rich) debt when (non-rich) incomes are held constant and state and year effects (δ_s, δ_t) are controlled for. Table 1 reports the results. Columns (1) and (2) show that an increase in lagged top incomes has a statistically significant positive effect on total household debt—for both the population and non-rich households. Holding non-rich incomes constant, an increase in lagged top incomes by 1% is associated with an increase in non-rich debt by over 0.2%. Columns (3) and (4) show the results for mortgage debt. The effect on mortgage debt is even stronger. In contrast, columns (5) and (6) show that top incomes do not have a positive effect on non-mortgage debt. If anything, the relationship is negative.

[8]We use lagged top incomes for two reasons. First, building houses takes time. Second, if non-rich households *keep up with the richer Joneses*, they will only react once they see the houses of the rich. We use the second lag of top incomes, but results are robust to using lags greater than two.

Figure 4: Residualized Household Debt and Lagged Top Incomes



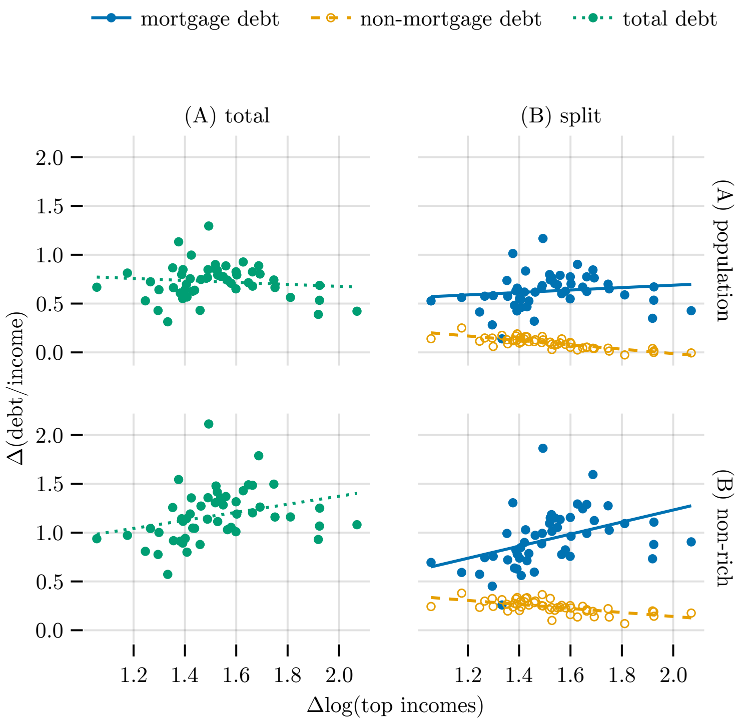
Notes: This figure shows the relationship between log debt (total, mortgage, non-mortgage) of all households and non-rich households and the second lag of log average top 10% incomes conditional on state and year fixed effects and non-rich income. All variables are residualized using state and year fixed effects. The slope of the regression line is the OLS estimate of β reported in Table 1. The figure shows averages in 20 equally sized bins of the x-variable.

Figure 4 visualizes the regression results using binned scatter plots of residualized log debt against residualized lagged log top incomes. The slope of the fit is equal to β in equation (1). The regression model is able to capture a substantial amount of the relationship of household debt and top incomes after accounting for fixed effects and own income.

Figure 5 shows the raw relationship between the long-run changes (1982 to 2007) in top incomes and household debt-to-income ratios. While there seems to be no significant relationship between top incomes and the debt-to-income ratio in the total population, there is a significant positive relationship between the change in top incomes and the change in the debt-to-income ratio of *non-rich* households.[9] Note also that the slope in panel A is pulled towards zero by four states with exceptionally high income growth: New York, Massachussetts, Connecticut and the District of Columbia. Figure 19 in Appendix A shows the slopes with and without these four states. Importantly, the picture is highly consistent with the above fixed effect regression results when looking at the relationship between top incomes and *non-rich* households' debt-to-income ratio irrespective of whether we include the four states with exceptionally high top income growth. Recall that this is

[9]Table 6 in the appendix reports the results of the bivariate regressions depicted in Figure 5.

Figure 5: Long-Run Changes in Household Debt and Top Incomes



Notes: This figure shows the relationship between the change in debt-to-income (total, mortgage, non-mortgage) of all households (population) and non-rich households (bottom 90% of the income distribution) and the change in the log of average top 10% incomes between 1982 and 2007 across US states.

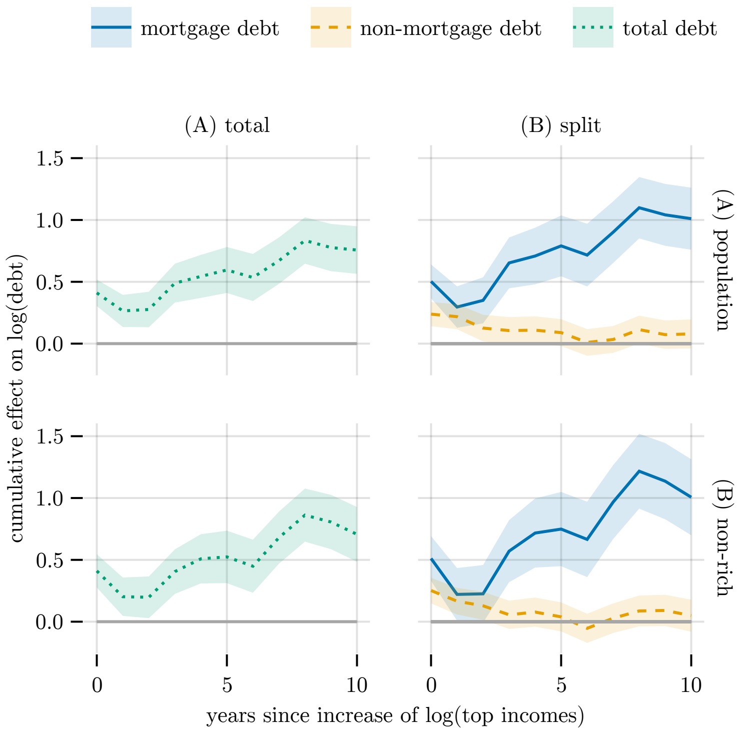
a key prediction of the KURJ mechanism. In addition, Figure 18 in the appendix shows that this positive relationship between long-run changes in top incomes and non-rich debt also holds for the middle 40% (P50 to P90) and bottom 50% of the income distribution.

2.3 Top Incomes and Household Debt: Dynamic Effects

To complement the two-way fixed effect regressions, we now analyze the dynamic response of household debt to changes in top incomes. In particular, we estimate how household debt changes from time $t-1$ to $t+h$ in response to a change in top incomes from $t-1$ to t using local projections of the form

$$\begin{aligned} \Delta^{h+1}\log(\text{debt}_{s,t+h}) &= \alpha^h + \beta^h \Delta\log(\text{top incomes}_{st}) + \delta_t^h + \\ &\sum_{k=1}^{3}\Big(\gamma_k^h \log(\text{debt}^g_{s,t-k}) + \phi_k^h \log(\text{top incomes}_{s,t-k})\Big) + \epsilon_{st}^h \qquad (2) \end{aligned}$$

Figure 6: Dynamic Effects of Top Incomes on Household Debt



Notes: This figure shows the cumulative effect of a 1% change in top 10% incomes on total, mortgage and non-mortgage debt in the population and among the non-rich estimated from equation 2. The confidence bands are constructed using a significance level of 5%.

for each $h \in \{0, \ldots, 10\}$, where

$$\Delta^{h+1} \log(\text{debt}_{s,t+h}) \equiv \log(\text{debt}^g_{s,t+h}) - \log(\text{debt}^g_{s,t-1})$$
$$\Delta \log(\text{top incomes}_{st}) \equiv \log(\text{top incomes}_{s,t}) - \log(\text{top incomes}_{s,t-1})$$

The coefficients β^h give us the cumulative %-change in non-rich debt that is induced by a one-time change in top-incomes by 1%. By adding past debt and inequality measures as controls, specification (2) essentially compares states with the same pretrends in debt and top incomes, but where one state experiences a stronger increase in top incomes from $t-1$ to t.

Figure 6 plots the estimated impulse response function for total, mortgage and non-mortgage debt both in the population and among the non-rich. Consistent with the previous fixed effect regressions, top incomes substantially drive up mortgage debt over the following ten years while non-mortgage debt remains roughly constant.[10] For the non-rich, a 10% increase in top incomes from $t-1$ to t translates into a persistent increase in mortgage debt of roughly 10% after ten years. For non-mortgage debt, there are no (persistent) effects.

[10] Controlling for lags of non-rich income or total income does not change the results.

Overall, these results not only show that rising top incomes are associated with rising debt-to-income ratios in the population and in particular among the non-rich, they also show that housing plays a central role in the transmission. While the aggregate (i.e. nation-wide) non-mortgage-debt-to-income ratio has gone up along with top incomes from 1980 to 2007, this link does not hold up on the state-level. This asymmetry between mortgage and non-mortgage debt is consistent with social comparisons in housing given that housing comparisons have at least some spatial bias. Even in the presence of modern communication technology, the local rich are arguable more visible and thus impose a greater status externality on households in the same state compared to other households across the country.

2.4 Top Incomes and House Prices

Having documented that state-level top incomes are associated with a subsequent increase in (non-rich) household mortgage debt, we now ask how state-level house prices react to rising top incomes. We estimate the following fixed effect regression

$$\log(\mathrm{HPI}_{st}) = \alpha + \beta \log(\text{top incomes}_{s,t-1}) + \gamma \log(\text{incomes}_{st}) + \delta_s + \delta_t + \epsilon_{st} \tag{3}$$

for two measures of state-level house prices. We deflate state-level nominal house prices from the Federal Housing Finance Agency by consumer prices. In Columns (1) and (2), we use novel state-level consumer price data from Hazell et al. (2020) to construct real state-level house prices. Unfortunately, these data are only available for a shorter sub-sample.[11] That is why we also report house prices deflated by nation-wide consumer prices in Columns (3) and (4).[12] The results are shown in Table 2. For both house price measures, we find that lagged top incomes have a statistically significant effect on house prices when total or non-rich income is held fixed.

Figure 7 shows the dynamic effect of top incomes on house prices estimated using the following estimation equation:

$$\begin{aligned}\Delta^{h+1}\log(\mathrm{HPI}_{s,t+h}) = \alpha^h + \beta^h \Delta \log(\text{top incomes}_{st}) + \delta_t^h + \sum_{k=1}^{3} \gamma_k^h \log(\mathrm{HPI}^g_{s,t-k}) \\ + \sum_{k=1}^{3} \phi_k^h \log(\text{top incomes}_{s,t-k}) + \sum_{k=1}^{3} \psi_k^h \log(\text{total incomes}_{s,t-k}) + \epsilon_{st}^h\end{aligned}$$

We find a statistically significant hump-shaped response of house prices to an increase in top incomes. Without state-level CPI data but for the full sample (left sub-figure), we find that house prices are up by 1% after ten years following a 1% increase in top incomes. When using state-level CPI data for a subset of states and years, the effect is slightly lower, but the overall pattern is very similar.

[11] The state-level CPI data are only available starting in 1979 for only 21 states, starting in 1988 for another 13 and not available at all for the remaining states. Nominal state-level house price data are available for all states starting in 1979.

[12] The use of year fixed effects takes out the aggregate trend in consumer prices.

TABLE 2: Top Incomes and House Prices: Fixed Effects Regressions

	log(real house price$_{s,t}$)			
	State FE-level CPI		country-level CPI	
	(1)	(2)	(3)	(4)
log(top incomes$_{s,t-2}$)	0.589***	0.440***	0.686***	0.570***
	(0.112)	(0.108)	(0.116)	(0.117)
log(bottom incomes$_{s,t}$)	0.580***		0.517***	
	(0.131)		(0.107)	
log(total incomes$_{s,t}$)		0.678***		0.529***
		(0.097)		(0.079)
State FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	OLS
N	793	793	1,428	1,428
R^2	0.925	0.924	0.947	0.945

Notes: This table shows the results of regression model in equation (3). Robust standard errors in parentheses. The stars indicate the range of the p value: $^{***} \leq 0.01 \leq {}^{**} \leq 0.05 \leq {}^{*} \leq 0.1$. Data: DINA, IRS.

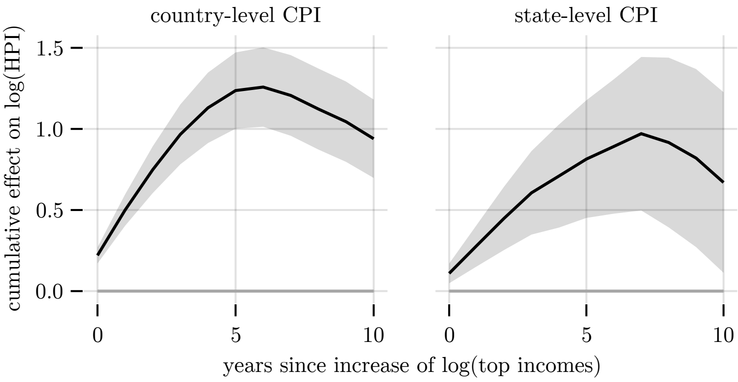
The result that growing top incomes trigger not only an increase in mortgage debt but also an increase in house prices points to a role for housing-demand effects to complement credit-supply effects in order to understand the boom in household debt.

3 Model

Motivated by these empirical findings that call for a housing-demand channel, we now evaluate whether the combination of rising inequality and *Keeping up with the richer Joneses* can help rationalize the US mortgage and housing boom. To that end, we incorporate social comparisons into an otherwise standard macroeconomic model of housing. This section describes our model and sections 4 and 6 present our analytical and quantitative results.

Our model is a dynamic, incomplete markets general equilibrium model similar to the "canonical macroeconomic model with housing" in Piazzesi and Schneider (2016). We formulate our model in continuous time to take advantage of the fast solution methods of Achdou, Han, Lasry, Lions, and Moll (2021, in particular Section 4.3). We build our model with two aims in mind. First, we want to illustrate how rising top-incomes and social comparisons can lead to rising debt levels across the whole income distribution. And second, we want to quantify the effect of this channel on the increase in aggregate mortgage debt and house prices from 1980 to 2007.

FIGURE 7: Dynamic Effects of Top Incomes on House Prices



Notes: This figure shows the cumulative effect of a 1% change in top 10% incomes on house prices estimated from equation 2.4. The confidence bands are constructed using a significance level of 5%.

3.1 Setup

Time is continuous and runs forever. There is a continuum of households that differ in their realizations of the earnings process. Households are indexed by their current portfolio holdings (a_t, h_t), where a_t denotes financial wealth and h_t denotes the housing stock, and their pre-tax earnings y_t. They supply labor inelastically to the non-durable consumption good and housing construction sectors. The financial intermediary collects households' savings and extends mortgages subject to a collateral constraint. The state of the economy is the joint distribution $\mu_t(\mathrm{d}a, \mathrm{d}h, \mathrm{d}y)$. There is no aggregate uncertainty.

3.2 Households

Households die at an exogenous mortality rate $m > 0$. The wealth of the deceased is redistributed to surviving individuals in proportion to their asset holdings (perfect annuity markets). Dead households are replaced by newborn households with zero initial wealth and earnings drawn from its ergodic distribution.[13] Households derive utility from a non-durable consumption good c and housing status s. They supply labor inelastically and receive earnings y. After-tax disposible earnings are given by

$$\tilde{y}_t = y_t - T(y_t),$$

where T is the tax function. Households choose streams of consumption $c_t > 0$, housing $h_t > 0$ and assets $a_t \in \mathbb{R}$ to maximize their expected discounted lifetime utility

$$\mathbb{E}_0 \int_0^\infty e^{-(\rho+m)t} \frac{\Big((1-\xi)c_t^\varepsilon + \xi s(h_t, \bar{h}_t)^\varepsilon\Big)^{\frac{1-\gamma}{\varepsilon}}}{1-\gamma} \mathrm{d}t,$$

[13]This follows Kaplan, Moll, and Violante (2018).

where $\rho \geq 0$ is the discount rate and the expectation is taken over realizations of idiosyncratic earnings shocks. $1/\gamma > 0$ is the inter-temporal elasticity of substitution, $1/(1-\varepsilon) > 0$ is the intra-temporal elasticity of substitution between consumption and housing status and $\xi \in (0,1)$ is the relative utility-weight for housing status.

A household's utility from housing is a function of the *housing status* $s(h, \bar{h})$. Housing status increases in the household's housing stock h and decreases in reference housing $\bar{h}$ which is a function of the equilibrium distribution of housing as introduced in the next section.

Housing is both a consumption good and an asset. It is modeled as a homogenous, divisible good. As such, h represents a one-dimensional measure of housing quality (including size, location and amenities). An agent's housing stock depreciates at rate δ and can be adjusted frictionlessly.[14] Home improvements and maintenance expenditures x_t have the same price as housing (p) and go into the value of the housing stock one for one.

Households can save ($a > 0$) and borrow ($a < 0$) at the equilibrium interest rate r. Borrowers must post their house as collateral to satisfy an exogenous collateral constraint. The collateral constraint pins down the maximum possible loan-to-value ratio ω.

Households' assets evolve according to

$$\begin{aligned} \dot{a}_t &= \tilde{y}_t + r_t a_t - c_t - p_t x_t, \\ \dot{h}_t &= -\delta h_t + x_t, \end{aligned}$$

subject to the constraints

$$\begin{aligned} a_t &\geq -\omega p_t h_t, \\ h_t &> 0. \end{aligned} \tag{4}$$

3.3 Social Comparisons

We build on the macroeconomic literature (e.g. Abel, 1990; Gali, 1994; Campbell and Cochrane, 1999; Ljungqvist and Uhlig, 2000) on keeping up with the Joneses and bring it closer to the empirical evidence. These papers feature representative agent models with one good and one asset. Agents compare themselves in the single consumption good, and their reference measure is the average consumption in the economy.[15]

We depart from this literature in two ways. First, we assume that households compare themselves only in their houses. This captures that people compare themselves only in conspicuous goods and that housing is one of the most important conspicuous goods—both in terms of visibility and expenditure share (e.g. Solnick and Hemenway, 2005; Bertrand and Morse, 2016).

Second, we allow the reference measure to be a function of the distribution of houses (and not necessarily its mean): $\bar{h}_i = \bar{h}_i(\mu_h)$. This reflects that the comparison motive is asymmetric, being strongest (and best documented) with respect to the rich (e.g. Clark

[14] Frictionless adjustment is justified, because we will be comparing long-run changes (over a period of 27 years).

[15] In equilibrium the reference measure has to be equal to the optimal choice of the representative agent.

and Senik, 2010; Ferrer-i-Carbonell, 2005; Card et al., 2012, on self-reported well-being). People buy bigger cars when their neighbors win in the lottery (Kuhn et al., 2011); non-rich move their expenditures to visible goods (such as housing) when top incomes rise in their state (Bertrand and Morse, 2016); and construction of very big houses leads to substantially lower levels of self-reported housing satisfaction for other residents in the same area—while the construction of small houses does not (Bellet, 2019).

For our analytical results we assume that $\bar{h}$ is a weighted mean of the housing distribution and use $s(h, \bar{h}) = h - \phi\bar{h}$ for tractability. For the quantitative results, we set $\bar{h}$ to the 90th percentile of the housing distribution and use $s(h, \bar{h}) = \frac{h}{\bar{h}^{\phi}}$ based on empirical evidence (see Section 5).

3.4 Pre-Tax Earnings Process

In our main experiment, we want to adjust life-time (permanent) income inequality independently of income risk to capture the way income inequality has changed over time. We follow Guvenen, Karahan, Ozkan, and Song (2021), who estimate a pre-tax earnings process on administrative earnings data. The process consists of individual fixed effects ($\tilde{\alpha}_i \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha)$), a persistent jump-drift process (z_{it}), a transitory jump-drift process (ϵ_{it}), and heterogeneous non-employment shocks ($\nu_{it} \in \{0, 1\}$).[16] We translate their estimated process to continuous time. Heterogeneity in $\tilde{\alpha}_i$ represents fixed ex-ante differences in earnings ability which is an important source of life-time inequality. If employed, individual pre-tax earnings are given by

$$y_{it}^{\text{pot}} = \exp(\tilde{\alpha}^i + z_{it} + \epsilon_{it}).$$

We will refer to y^{pot} as *potential earnings*. The actual pre-tax earnings (taking into account unemployment) are

$$y_{it} = (1 - \nu_{it}) y_{it}^{\text{pot}}.$$

See Appendix D for more details on the earnings process.

3.5 Production

There are two competitive production sectors producing the non-durable consumption good c and new housing investment I_h, respectively. Following Kaplan et al. (2020), there is no productive capital in this economy.

Non-Durable Consumption Sector The final consumption good is produced using a linear production function

$$Y_c = N_c$$

[16] We use version (7), where we take out the deterministic life-cycle profile. The only component that this version does not have are differences in deterministic income growth rates.

where N_c are units of labor working in the consumption good sector. As total labor supply is normalized to one, N_c is also the share of total labor working in this sector. The equilibrium wage per unit of labor is pinned down at $w = 1$.[17]

Construction Sector We model the housing sector following Kaplan et al. (2020) and Favilukis et al. (2017). Developers produce housing investment I_h from labor $N_h = 1 - N_c$ and buildable land $\bar{L}$, $I_h = (\Theta N_h)^{\alpha}(\bar{L})^{1-\alpha}$ with $\alpha \in (0, 1)$. Each period, the government issues new permits equivalent to $\bar{L}$ units of land, and these are sold at a competitive market price to developers. A developer solves

$$\max_{N_h} p_t I_h - w N_h \quad \text{s.t. } I_h = N_h^{\alpha} \bar{L}^{1-\alpha}$$

In equilibrium, this yields the following expression for optimal housing investment

$$I_h = (\alpha p)^{\frac{\alpha}{1-\alpha}} \bar{L}$$

which implies a price elasticity of aggregate housing supply of $\frac{\alpha}{1-\alpha}$.

3.6 Financial Markets

The financial intermediary collects savings from households and issues mortgages to households. Lending is limited by the households' exogenous collateral constraint (4).

In addition, the intermediary has an exogenous net asset position with the rest of the world a_t^S. The equilibrium interest ensures that bank profits are zero and the asset market clears,

$$\int a_t(a, h, y) \mathrm{d}\mu_t = a_t^S. \tag{5}$$

3.7 Stationary Equilibrium

A stationary equilibrium is a joint distribution $\mu(a, h, y)$, policy functions $c(a, h, y, \bar{h})$, $x(a, h, y, \bar{h})$, $h(a, h, y, \bar{h})$, $a(a, h, y, \bar{h})$, prices (p, r) and a reference measure $\bar{h}$ satisfying the following conditions

- Policy functions are consistent with agents' optimal choices $(c_t, h_t, a_t)_{t>0}$ given incomes $(y_t)_{t>0}$, prices p, r and the reference measure $\bar{h}$.
- Housing investment is such that the construction sector maximizes profits.
- $\mu(a, h, y)$ is stationary. That is, if the economy starts at μ, it will stay there.
- Asset market clears (5) and housing investment equals housing production $\int x(a, h, y) d\mu = I_h$.
- The reference measure is consistent with choices: $\bar{h} = \bar{h}(\mu)$.

[17] Neither labor supply nor the wage appear in the earnings process, because there is no aggregate risk, households inelastically supply one unit of labor, and the wage is equal to 1.

4 Analytical Results

In this section we use a stylized version of the model described in section 3 to illustrate analytically how rising top incomes can lead to rising mortgage levels across the whole income distribution via social comparisons. In Proposition 1 we provide formulas for optimal housing and consumption, as functions of their permanent incomes, and the permanent incomes of the direct and indirect reference groups. In Proposition 2 we show that optimal debt is increasing in the incomes of the direct and indirect reference groups. In Proposition 3 we show that the impact of rising incomes $\tilde{y}_i$ on aggregate debt is increasing in type i's *popularity*. In Corollary 1 we show that total debt-to-income is increasing in top incomes if at least one person compares themselves to the rich. In Corollary 2 we show that under Cobb-Douglas aggregation ($\varepsilon = 0$), these results hold even under housing market clearing because they are independent of house prices p. In Corollary 3 we show that these results crucially depend on the fact the status good h is durable.

The assumptions needed to obtain tractability are that there is no idiosyncratic income risk; that the social status function is linear; and that the interest rate equals the discount rate (all of these assumptions are relaxed in the following sections).

Assumption 1. $r = \rho$.

Further, we assume that there is a finite number of types of households $i \in \{1, \dots, N\}$. Agents vary by their initial endowments a_0 and flow disposable income $\tilde{y}$.

Assumption 2. Flow income $\tilde{y}_i$ is deterministic and constant over time, but varies across types i.

Without loss of generality, we assume that types are ordered by their permanent income $\mathcal{Y}_i = ra_0^i + \tilde{y}_i$,

$$\mathcal{Y}_1 \leq \mathcal{Y}_2 \leq \dots \leq \mathcal{Y}_N.$$

We use bold variables to denote the vector variables for each type using the above ordering, e.g. $\boldsymbol{h} = (h_1, \dots, h_N)^T$.

Assumption 3 (Tractable social comparisons)**.** The status function $s(h, \bar{h}) = h - \phi\bar{h}$ is linear and the reference measure $\bar{h}_i = \sum_{j \neq i} g_{ij} h_j$ is a weighted sum of other agent's housing stock (we assume $g_{ij} \geq 0$).

Note, that we can write the vector of reference measures as $\bar{\boldsymbol{h}} = (\bar{h}_1, \dots, \bar{h}_N)^T = G \cdot \boldsymbol{h} := (g_{ij})(h_i)$. The matrix G can be interpreted as the adjacency matrix of the network of types capturing the comparison links between agents of each type. g_{ij} measures how strongly agent i cares about agent j.

We further require the comparisons to satisfy the following regularity condition.

Assumption 4. The Leontief inverse $(I - \phi G)^{-1}$ exists and is equal to $\sum_{i=0}^{\infty} \phi^i G^i$ for ϕ from Assumption 3.

This assumption is not very strong. This assumption is satisfied whenever the power of the matrix converges, $G^i \to G^\infty$. For example, if G represents a Markov chain with a stationary distribution or if G is nilpotent.

4.1 Characterization of the Partial Equilibrium

We solve for a simplified version of the equilibrium in Section 3.7. Agents solve their optimization problem given prices and the reference measure; the reference measure is consistent; but for now, we don't require market clearing. We use a lifetime budget constraint instead of the implicit transversality condition.

Households optimal decisions are given in the following proposition.

Proposition 1. *Under assumptions 1, 2, 3 and 4 the optimal choices* $\boldsymbol{h} = (h_1, \ldots, h_N)^T$ *and* $\boldsymbol{a} = (a_1, \ldots, a_N)^T$ *are given by*

$$\boldsymbol{h} = \Big(\sum_{i=0}^{\infty} (\kappa_1 \phi G)^i\Big) \kappa_2 \boldsymbol{\mathcal{Y}}.$$

$$-r\boldsymbol{a} = \tilde{\boldsymbol{y}} - \kappa_3 \boldsymbol{\mathcal{Y}} + (1-\kappa_3)\Big(\sum_{i=1}^{\infty} (\kappa_1 \phi G)^i\Big) \boldsymbol{\mathcal{Y}} \tag{6}$$

where $\kappa_1 = \frac{1}{\frac{p(r+\delta)}{\kappa_0}+1} \in (0,1)$, $\kappa_2 = \frac{\kappa_1}{\kappa_0}$, $\kappa_3 = \frac{1}{1+\frac{pr}{\delta p + \kappa_0}} \in (0,1)$ *and* $\kappa_0 = \Big((r+\delta)\frac{1-\xi}{\xi}p\Big)^{\frac{1}{1-\varepsilon}}$.

Proof. See appendix C.2. □

Households' choices depend on a weighted average of the permanent incomes of their (direct and indirect) reference groups. The weights are positive, whenever there is a direct or indirect social link between those agents. This is captured by the *income-weighted Bonacich centrality*, $B = \sum_{i=0}^{\infty}(C_1 \phi G)^i \boldsymbol{\mathcal{Y}}$. If the weight B_{ij} is positive, household j's lifetime income affects household i's choices. This is the case whenever j is in i's reference group (there is a direct link $g_{ij} > 0$), or if j is in the reference group of some agent k who is in the reference group of agent i (there is an indirect link of length two, $g_{ik} g_{kj} > 0$) or if there is any other indirect link ($\prod_{n=1}^{N-1} g_{\ell_n, \ell_{n+1}}$ where $\ell_1 = i$ and $\ell_{N-1} = j$).

These results are reminiscent of those in Ballester et al. (2006). They showed that the unique Nash equilibrium in a large class of network games is proportional to the (standard) Bonacich centrality.

4.2 Comparative Statics

First, we show that optimal debt and optimal housing are increasing in incomes of the direct and indirect comparison groups.

Proposition 2. *For each type j in i's reference group (that is, $g_{ij} > 0$) and for each k that is in the reference group of the reference group (etc.) of i (that is, there is $j_1, j_2, \ldots, j_n$ such that $g_{ij_1} g_{j_1 j_2} \cdots g_{j_{n-1} j_n} g_{j_n k} > 0$), then h_i is increasing and a_i is decreasing in $\mathcal{Y}_j$ (or $\mathcal{Y}_k$).*

Proof. G is non-negative, so $\sum_i c^i G^i$ is non-negative for all $c \geq 0$. From the definition of the Leontief inverse, being the discounted sum of direct and indirect links it follows,

$$\frac{\partial h_i}{\partial \tilde{y}_j} > \kappa_2 \kappa_1 \phi g_{ij} > 0 \quad \text{and} \quad \frac{\partial h_i}{\partial \tilde{y}_k} > \kappa_2 (\kappa_1 \phi)^{n-1} g_{ij_1} g_{j_1 j_2} \cdots g_{j_{n-1} j_n} g_{j_n k} > 0.$$

Similarly

$$-\frac{\partial a_i}{\partial \tilde{y}_j} > (1-\kappa_3)\kappa_1\phi g_{ij} > 0 \quad \text{and} \quad -\frac{\partial a_i}{\partial \tilde{y}_k} > (1-\kappa_3)(\kappa_1\phi)^{n-1}\phi g_{ij_1}g_{j_1j_2}\cdots g_{j_{n-1}j_n}g_{j_nk} > 0.$$

□

Agent A's debt increases if agent B's lifetime income increases—as long as there is a direct or indirect link from A to B. That link exists, if agent A cares about agent B, or if agent A cares about some agent C who cares about agent B.

Second, we show how aggregage housing and debt react to changes in type j's income $\mathcal{Y}_j$. We first define the popularity of a type.

Definition 1 (Popularity)**.** We define the vector of popularities as

$$\boldsymbol{b}^T = \mathbf{1}^T \sum_{i=1}^{\infty} (\kappa_1 \phi G)^i,$$

and type i's popularity b_i as the ith component of $\boldsymbol{b}$.

The popularity is the sum of all paths that end at individual i. It measures how many agents compare themselves with i (directly and indirectly) and how strongly they do. The popularity of a type is crucial in determining how strongly their income will affect economic aggregates.

Proposition 3. *The impact of a change in type j's on aggregate housing and aggregate debt is proportional to its popularity.*

$$\frac{\partial}{\partial \tilde{y}_j}\sum_i h_i = \kappa_2(1+b_j)$$
$$\frac{\partial}{\partial \tilde{y}_j}\sum_i r a_i = (1-\kappa_3)(1+b_j).$$

Proof. Take the expressions from proposition 1 and plug in the definitions for $\mathcal{Y}$ and b (Definition 1), aggregate housing can be written as $\sum_{i=1}^N h_i = \kappa_2 \sum_{i=1}^N (1+b_i)(\tilde{y}_i + r a_0^i)$ and aggregate debt can be written as $-\sum_{i=1}^N r a_i = (1-\kappa_3)\sum \tilde{y}_i - \kappa_3 \sum a_0^i + (1-\kappa_3)\sum_{i=1}^N b_i(\tilde{y}_i + r a_0^i)$. The derivatives follow immediately. □

Corollary 1. *If all types $i \neq j$ are connected to agent j and $\tilde{y}_j$ increases, then debt-to-income increases for all types $i \neq j$.*

Proof. By Proposition 2 debt of types $i \neq j$ increases, while their income is unchanged. It follows that debt-to-income rises. □

Corollary 2. *Under Cobb-Douglas aggregation, the results for a in Propositions 1, 2 and 3 are independent of house prices.*

Proof. Under Cobb-Douglas κ_0 is divisible by p. This means that p cancels in κ_1 and κ_3. Thus, all p cancel in the expression for a in Proposition 1 and consequently doesn't show up in the respective expressions in Propositions 2 and 3. □

The results on optimal debt in Propositions 2 and 3 and Corollary 1 break down if houses are non-durable. For any small time interval Δ, the depreciation rate has to be $\delta = \frac{1}{\Delta}$, so that the housing stock depreciates immediately,

$$(1 - \Delta\delta)h_t = 0.$$

Note the familiar special case when $\Delta = 1$ ("discrete time"), then the depreciation rate must be $\delta = 1$ for goods to be non-durable.

Corollary 3. *When houses are non-durable, optimal debt does not depend on others' incomes.*

Proof. In continuous time $\Delta \to 0$, so $\delta \to \infty$. It can be easily seen that $\kappa_3 \to 1$ as $\delta \to \infty$, thus $(1 - \kappa_3) \to 0$. Since all other terms in expression (6) are bounded, the part containing the Leontief inverse vanishes and becomes $-r\boldsymbol{a} = \tilde{\boldsymbol{y}} - \boldsymbol{\mathcal{Y}} = -r\boldsymbol{a}_0$. □

Note that this result does not depend on continuous time. The same result works in a discrete time version of the model, where $\Delta = \delta = 1$ and no limit argument is involved.

4.3 How Rising Top Incomes Fuel the Mortgage Boom: Intuition

It is at the heart of the mechanism that there is a complementarity between a household's housing stock and their reference measure. When top incomes $\mathcal{Y}_N$ rise, households of type N will improve (or upsize) their housing stock h_N, increasing the reference measure $\bar{h}_i$ for all types i that care about type N directly or indirectly. Each of these agents will optimally substitute durable, status-enhancing housing for non-durable status neutral consumption.

For debt to be affected it is key that the status good is durable and the status-neutral good is non-durable (see Corollary 3). Households want their stock of the durable good to be constant over time. Therefore, they need to pay for the entire house ph upfront and only replace the depreciation δph in the future. In other words, households need to shift some of their lifetime income forward to finance their house and take on mortgage debt to achieve that. The greater the value of the house, the bigger is the necessary mortgage.

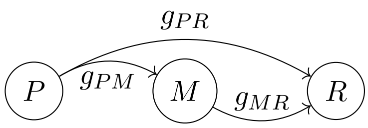
4.4 Implications for Renters and Non-Mortage debt

We can use the limit case of non-durable housing to analyze the model implications for *non-mortgage debt*. If housing is non-durable, then housing services are essentially rented from real estate owners outside the model. As there is no house to finance, debt should now be interpreted as unsecured debt, smoothing out variations in earnings (a_0 vs y).

When top incomes rise and the rich scale up their (rental) housing, the other groups still substitute status enhancing housing services for the status-neutral consumption good, but there is *no effect on debt* (see Corollary 3).

4.5 Example: Upward Comparisons with Three Types of Agents

We now illustrate the results for the simple case of three types of agents, poor P, middle class M, and rich R. The poor type compares himself with both other types, the middle type compares himself only with the rich type, and the rich type not at all. Figure 8 shows the corresponding graph and its adjacency matrix.



(A) The Graph

$$G = \begin{array}{c c} & \begin{array}{ccc} P & M & R \end{array} \\ \begin{array}{c} P \\ M \\ R \end{array} & \begin{pmatrix} 0 & g_{PM} & g_{PR} \\ 0 & 0 & g_{MR} \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

(B) The Adjacency Matrix

FIGURE 8: The social network structure with three types, assuming upward comparisons. The network can be represented as a graph and as its adjacency matrix.

Since G is a triangular matrix with only zeros on the diagonal, it is nilpotent ($G^3 = \mathbf{0}$), and thus the Leontief inverse exists.

$$G^2 = \begin{array}{c c} & \begin{array}{ccc} P & M & R \end{array} \\ \begin{array}{c} P \\ M \\ R \end{array} & \begin{pmatrix} 0 & 0 & g_{PM}g_{MR} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}, \quad G^3 = \mathbf{0}$$

The matrix G^2 counts the paths of length 2. In our example there is only one such path—from type P to type R. Defining $\tilde{\phi} = \kappa_1 \phi$, the vector of Bonacich centralities is given by

$$\sum_{i=0}^{\infty} \alpha^i G^i = I + \sum_{i=1}^{2} \alpha^i G^i = I + \begin{pmatrix} 0 & \alpha \cdot g_{PM} & \alpha \cdot g_{PR} + \alpha^2 \cdot g_{PM} \cdot g_{MR} \\ 0 & 0 & \alpha \cdot g_{MR} \\ 0 & 0 & 0 \end{pmatrix}$$

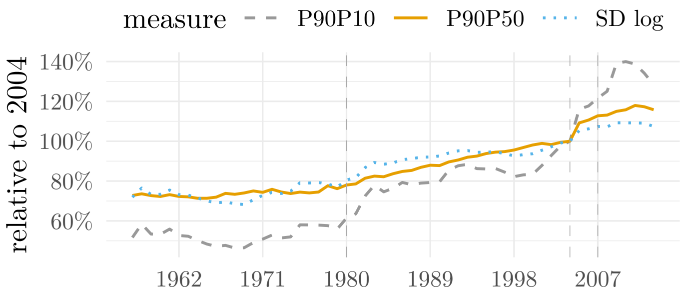
The partial equilibrium choices for housing and debt are now given by

$$\begin{pmatrix} h_P \\ h_M \\ h_R \end{pmatrix} = \kappa_2 \begin{pmatrix} 1 & \tilde{\phi} \cdot g_{PM} & \tilde{\phi} \cdot g_{PR} + \tilde{\phi}^2 \cdot g_{PM} \cdot g_{MR} \\ 0 & 1 & \tilde{\phi} \cdot g_{MR} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{Y}_P \\ \mathcal{Y}_M \\ \mathcal{Y}_R \end{pmatrix}$$

$$-r \begin{pmatrix} a_P \\ a_M \\ a_R \end{pmatrix} = \tilde{\boldsymbol{y}} - \kappa_3 \boldsymbol{\mathcal{Y}} + (1 - \kappa_3) \begin{pmatrix} 0 & \tilde{\phi} \cdot g_{PM} & \tilde{\phi} \cdot g_{PR} + \tilde{\phi}^2 \cdot g_{PM} \cdot g_{MR} \\ 0 & 0 & \tilde{\phi} \cdot g_{MR} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{Y}_P \\ \mathcal{Y}_M \\ \mathcal{Y}_R \end{pmatrix}$$

An agent's housing choice increases linearly in own permanent income, $\mathcal{Y} = \tilde{y} + ra_0$, and on the permanent income of agents *in the reference group*. The poor agent's consumption increases through the direct links, but also indirect links (which are discounted more strongly). Agents' decisions to save or borrow depend on the ratio of initial wealth a_0 and income $\tilde{y}$. The higher the income relative to initial wealth, the greater the need to borrow.

FIGURE 9: US Earnings Distribution



Notes: This figure shows the change in the cross-sectional distribution of male earnings in the US. Vertical bars in 1980, 2004 and 2007. Source: Guvenen et al. (2018).

5 Parameterization

Now we return to the full model. We parameterize the model to be consistent with the aggregate relationships of mortgage debt, house value and income in the US at the beginning of the 1980s. We use the estimated income process from Guvenen et al. (2021) and assign eight other parameters externally. The remaining two parameters (the discount rate ρ and the utility weight of housing status ξ) are calibrated internally so that in general equilibrium the aggregate net-worth-to-income ratio and aggregate loan-to-value ratio match these aggregate moments in the 1983 Survey of Consumer Finances.

Income Process We translate the estimated income process from Guvenen et al. (2021) to continuous time. It has a permanent, a persistent and a transitory component and state-dependent unemployment risk. Guvenen et al. (2021) estimate it to data from the time period 1994–2013. In order to construct the income process for the baseline economy $\mathcal{E}$ (corresponding to the year 1980) we rescale the permanent component following evidence on the changes in the income distribution from Kopczuk et al. (2010), Guvenen, Ozkan, and Song (2014) and Guvenen et al. (2018).

The cross-sectional dispersion of incomes has increased substantially between 1980 and 2007. Figure 9 (taken from Guvenen et al., 2018, Figure 12) shows the variation of three common measures over time: the P90/P50 ratio, the P90/P10 ratio and the standard deviation of log-earnings. These changes in the variation of incomes can come from either component of the income process, or even a combination of them.

While there is no consensus yet,[18] as to which of those factors contributed how much, there is evidence that rising permanent inequality explains a substantial share in increased cross-sectional variation. Kopczuk et al. (2010, Figure V) find that almost all of the change in earnings variation came from increases in permanent inequality. This finding is supported by Guvenen et al. (2014, Figure 5) who show that the variances of earnings shocks have had a slight downward trend since 1980.

[18] Carr and Wiemers (2016, 2018) show that depending on data source, sample selection, and statistical model one can find substantial differences in the decomposition into risk and permanent inequality.

Given this evidence, we attribute all change in inequality to changes in permanent inequality (σ_α). In our income process, permanent income inequality is represented by the permanent component $\tilde{\alpha}$. So, given the discretized version of the process, we stretch the upper half of the $\tilde{\alpha}$-grid to match the changes in the cross-sectional P90/P50 ratio.

When translating the process to continuous time, we assume that shocks arrive on average once a year (instead of every year). Moreover, we replace the discrete time *iid* process by a jump-drift process (ϵ_{it}) that is re-centered around zero whenever a shock hits so that shocks do not accumulate. The mean reversion rate of the persistent process (z_{it}) is the negative log of the discrete time persistence parameter which preserves the same annual autocorrelation. The exit rate out of nonemployment is chosen to match the average duration of nonemployment stays in the discrete time process. As households in our infinite horizon model die at a constant rate, we remove all age-dependence by setting the age profile constant (to the value at the mean age $\bar{t}$).[19] Table 8 in the appendix shows all parameters of our continuous time earnings process.

We put the process on a discrete state space, using the approach of Kaplan et al. (2018). We discretize each component separately, obtaining continuous-time Markov chains[20] for the persistent and transitory components and combining them afterwards. Finally, we add the state-dependent non-employment risk.

Income Taxation We use the progressive income tax function from Heathcote, Storesletten, and Violante (2017),

$$T(y) = y - \tau_0 y^{1-\tau_1}.$$

If non-employed, households receive a fraction b of their potential earnings from unemployment insurance. Thus, the post-tax disposable income is given by

$$\tilde{y}_t = \begin{cases} y_{it}^{\text{pot}} - T(y_{it}^{\text{pot}}) & \text{if employed} \\ b y_{it}^{\text{pot}} & \text{otherwise.} \end{cases}$$

We follow Kaplan et al. (2020) in our choice of the parameters τ_0, τ_1. The progressivity parameter τ_1 is an estimate from Heathcote et al. (2017) and the scale parameter τ_0 is set to match the tax revenue from personal income tax and social security contribution as a share of GDP in 1980 (14.4%).[21] We set the replacement rate to 32%, matching average unemployment insurance benefits, as a fraction of average wage, as reported by the US Department of Labor.[22]

Preferences and Demographics The discount rate ρ and the utility weight of housing status ξ are internally calibrated to match the economy-wide mortgage-debt-to-income and loan-to-value ratios from the 1983 SCF. The interpretation of the utility weight ξ differs from other models, because ξ is the utility weight of housing status (not housing stock).

[19] This affects the mean of log earnings as well as the arrival rate of nonemployment shocks.
[20] Mostly called Poisson processes in the literature.
[21] Retrieved from `https://taxfoundation.org/federal-tax-revenue-source-1934-2018/`.
[22] Retrieved from `https://oui.doleta.gov/unemploy/DataDashboard.asp`.

TABLE 3: Baseline Parameters

Parameter description		Source	Value
Preferences			
ϕ	strength of keeping up motive	Bellet (2019)	0.7
ρ	discount rate	internal	0.02
ξ	utility weight of housing	internal	0.277
$\frac{1}{1-\varepsilon}$	intra-temporal elasticity of substitution	Flavin and Nakagawa (2008, AER)	0.15
γ	inverse intertemporal elasticity of substitution	standard	1.5
$\frac{1}{m}$	constant mortality rate	45 years worklife	45.0
Housing and financial technogy			
$\frac{\alpha}{1-\alpha}$	price elasticity of housing supply	Saiz (2010, QJE)	1.5
δ	depreciation rate of housing	Bureau of Economic Analysis	0.021
ω	maximum loan-to-value ratio	P95 of LTV	0.85
$a^S/\bar{y}$	exogenous net asst supply	cum. current account	-0.01
Taxation and Unemployment Insurance			
τ_0	level of taxes	internal	0.932
τ_1	progressivity	Heathcote et al. (2017)	0.15
b	replacement rate	Dept of Labor	0.32

The literature has not yet converged to a common value for the intratemporal elasticity of substitution $\frac{1}{1-\varepsilon}$. Estimates range from 0.13–0.24 (from structural models; e.g. Flavin and Nakagawa, 2008; Bajari, Chan, Krueger, and Miller, 2013) up to 1.25 (Ogaki and Reinhart, 1998; Piazzesi, Schneider, and Tuzel, 2007, using estimates from aggregate data). Many papers have picked parameters out of this range.[23] We follow the evidence from structurally estimated models and set the elasticity to 0.15.

The inverse intertemporal elasticity of substitution γ is set to the standard value 1.5. The constant annual mortality rate $m = 1/45$ is set to get an expected (working) lifetime of 45 years.

Social Comparisons For the status function we use a ratio-specification $s(h, \bar{h}) = \frac{h}{\bar{h}^{\phi}}$ as in Abel (1990). Bellet (2019) shows that this functional form captures the empirical finding that the utility loss from big houses decreases with own house size. Households with a medium sized house are more affected by top housing than households living in a small house.[24]

We define the reference measure as the 90^{th} percentile of the (endogenous) housing distribution, $\bar{h} = h_{\text{P90}}$. This follows Bellet (2019) who shows that households are only sensitive to changes in the top quintile of the house (size) distribution and strongest when the reference measure is defined as the 90^{th} percentile.[25]

[23] Garriga and Hedlund (2020) use 0.13, Garriga, Manuelli, and Peralta-Alva (2019) use 0.5, many papers use Cobb-Douglas (that is, an elasticity of 1.0, e.g. Berger, Guerrieri, Lorenzoni, and Vavra, 2018; Landvoigt, 2017) and Kaplan et al. (2020) use 1.25.

[24] Note that the more tractable linear specification $(h - \phi\bar{h})$ as used in Campbell and Cochrane (1999), Ljungqvist and Uhlig (2000) and Section 4 would imply the opposite relationship between own house size and comparison strength.

[25] See Figure 6 in Bellet (2019).

Table 4: Targeted Moments

Moment	Model	Data (80/83)
aggregate loan-to-value	0.24	0.24
aggregate networth-to-income	4.63	4.60
tax-revenue-to-income	0.14	0.14

The parameter ϕ pins down the strength of the comparison motive. It is the ratio of two utility elasticities

$$\phi = -\frac{\text{elasticity of utility w.r.t. } \bar{h}}{\text{elasticity of utility w.r.t. } h}.$$

If reference housing improves by 1%, then agents would have to improve their own house ϕ% to keep utility constant. Bellet (2019) estimates ϕ to be between 0.6 and 0.8 when setting $\bar{h}$ equal to the 90^{th} percentile of the housing distribution. We thus choose $\phi = 0.7$.[26] Note that Bellet (2019) estimates exactly this sensitivity using data on housing satisfaction which allows us to take his estimates without an intermediate indirect inference procedure. However, we note that this value for ϕ is likely an upper bound as our model does not have a spatial dimension. The implicit assumption is that the rise in top incomes and hence reference housing is equally spread across space.[27]

Technology and Financial Markets The construction technology parameter α is set to 0.6 so that the price elasticity of housing supply ($\frac{\alpha}{1-\alpha}$) equals 1.5, which is the median value across MSAs estimated by Saiz (2010). The maximum admissible loan-to-value ratio (ω) is set to 0.85, to match the 95th percentile of the LTV distribution in the SCF (Kaplan et al., 2020, use a similar approach for setting the debt-service-to-income constraint). Finally, we specify the exogenous net supply of assets a^S to match the net foreign debt position of the US. The net foreign debt position can be well approximated by the cumulative current account deficit of the US (Gourinchas et al., 2017), which was 1% of GDP in 1980 (see also Figure 13).

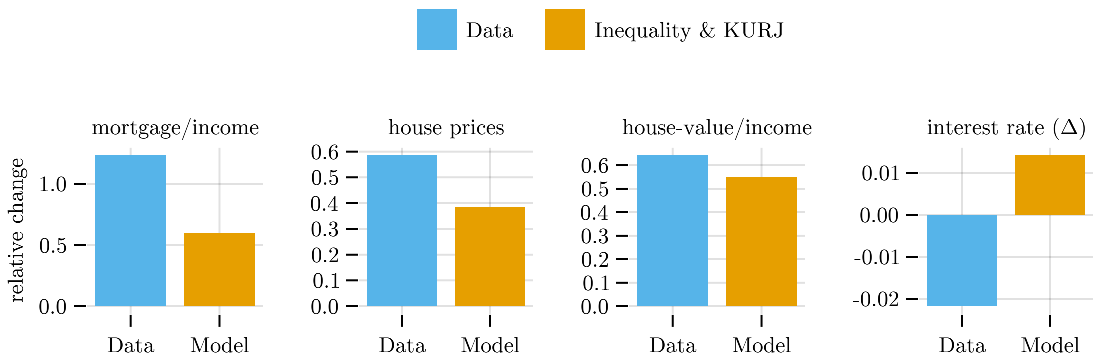
5.1 Internal Calibration and Model Fit

For the internal calibration we target the aggregate networth-to-income ratio (4.6) and the aggregate loan-to-value ratio (0.24) from the first wave of the Survey of Consumer Finances in 1983. We pick the utility weight of housing ξ and the the discount rate ρ so that simulated moments match their counterparts in the data. Table 4 shows that the model fits the data very well.

[26] See Table 2 in Bellet (2019)

[27] Bellet (2019) shows that the estimate of ϕ depends on the distance between the reference house and one's own house. The point estimate is greater or equal to 0.7 when a big house is built within a radius of up to 25 miles and close to zero for changes in the reference house that happen further away.

FIGURE 10: Steady State Effects – 1980 vs. 2007



Notes: This figure shows relative changes in aggregate variables between the steady states in 1980 and 2007 and the corresponding changes in the data. Data: DINA.

6 Quantitative Results

In this section we study how the model economy reacts to changes in the environment in the long-run. We compare the initial stationary equilibrium (corresponding to 1980) with the stationary equilibria where we increase income inequality to the level of 2007. Afterwards we set these results into perspective by comparing the effects of rising inequality and KURJ to the effect of increasing credit supply (Global Saving Glut). Lastly, we study the effects of both mechanisms combined.

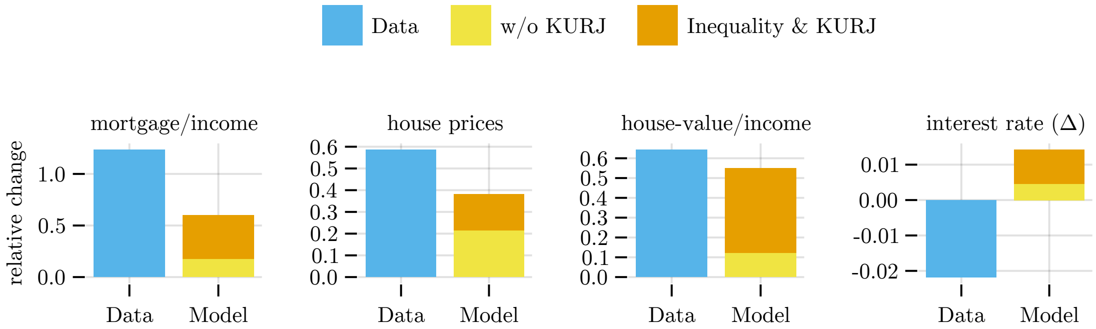
6.1 Effects of Rising Inequality & KURJ

We now move to the main experiment of the paper. We start from the steady-state calibrated to the U.S. economy in 1980. We then raise income inequality to match the level in 2007 and solve for the new general equilibrium. Before getting to the results, we describe how we model the increase in income inequality.

As we discuss in Section 5, the cross-sectional dispersion of income has increased substantially between 1980 and 2007. Given the evidence in Kopczuk et al. (2010) and Guvenen et al. (2014), we attribute this change in cross-sectional inequality to changes in permanent inequality. In our model permanent inequality is reflected by the standard deviation of the distribution of the permanent component σ_α of the income process. Hence, we increase σ_α to match the increase in the cross-sectional P90/P50 ratio.

Figure 10 summarizes the steady state comparison by comparing the changes in the mortgage-to-income ratio, the house-value-to-income ratio, house prices and interest rates from 1980 to 2007 in the model and the data. In the model, rising inequality and the presence of KURJ create both a mortgage boom and a house price boom. The left panel of Figure 10 shows that this mechanism generates an increase in the mortgage-to-income ratio of about 60%—roughly half of the increase that is observed in the data where the mortgage-to-income ratio went up by 123%. The increase in housing demand puts upward pressure on house prices which increase by 38% in the model. This corresponds to about 60% of the 58% house price increase observed in the data. The shift towards housing is

Figure 11: Decomposition of Steady State Effect – The Importance of KURJ



Notes: Comparison simulated changes in aggregate variables between the steady states in 1980 and 2007. "w/o KURJ" shows the changes when the reference measure $\bar{h}$ is kept fixed at level $\bar{h}_{1980}$ from the initial stationary equilibrium. Data: DINA.

also reflected in the house-value-to-income ratio which goes up by 55% versus 64% in the data.

The housing and mortgage boom in the model is the result of two channels. The first channel is the direct comparison effect. Households increase their housing demand to keep up with the new reference measure set by the rich. This channel raises the demand for mortgage debt through an increase in housing demand. Second, rising top incomes raise the demand for housing and thus house prices because the richer households want to live in bigger houses. Since housing and non-durable consumption are complements, the expenditure share on housing goes up for all households as the house price increases. This indirect effect works even in the absence of KURJ.

Figure 11 shows that the social comparison motive plays a quantitatively important role. In particular, we show how much of the overall effect can be obtained with rising inequality in the absence of KURJ.[28] Without KURJ, rising inequality generates a 20% increase in the mortgage-to-income ratio and a 22% increase in house prices. That implies that KURJ is required to generate most of the mortgage boom and almost half of the house price boom.

Note that the case without KURJ corresponds to setting $\phi = 0$. To the extent that our baseline parameter $\phi = 0.7$ (Bellet, 2019) is an upper bound, the graph also shows the range of effects for other choices of $\phi \in (0, 0.7]$. Importantly, even for intermediate values of ϕ, the contribution of KURJ is quantitatively important.

While rising top inequality and KURJ generate a sizable mortgage and housing boom, no one mechanism can explain all aspects of the data. Here, the increasing demand for mortgages results in a counterfactual prediction about the interest rate which increases in the model by 1.4 percentage points, but decreases in the data by 2 percentage points.

[28] Instead of re-calibrating the model with $s(h, \bar{h}) = h$ one can use that for a given reference measure $\bar{h}$ that is constant across the population, the initial equilibrium $\mathcal{E}$ is equivalent to a parameterization with $s(h, \bar{h}) = h$ and housing weight $\tilde{\xi}$ such that $\frac{\tilde{\xi}}{1-\tilde{\xi}} = \frac{\xi}{1-\xi} \frac{1}{\bar{h}^{\phi}}$. This holds because our specification of social comparisons, just re-weights the utility of housing and consumption.

Figure 12: Heterogeneity Across the Income Distribution

Notes: This figure shows the relative changes in the mortgage-to-income ratio, the house-value-to-income ratio and leverage (mortgage-to-house-value ratio) across the income distribution. Data: DINA.

Figure 12 shows the relative change in the mortgage-to-income ratio, the house-value-to-income ratio, and the loan-to-value ratio (leverage) across the income distribution. Both in the data and the model, the mortgage and housing boom spans the entire income distribution. For mortgages relative to income, the increase is especially strong in the bottom half of the income distribution. The model does a better job at the top of the income distribution relative to the bottom. The hump-shaped pattern is even more pronounced for the increase in house-values relative to incomes. Here, the model is broadly consistent with the heterogeneity across the income distribution as the increase is strongest in the middle and weakest at the top.

6.2 Comparison with the Global Saving Glut

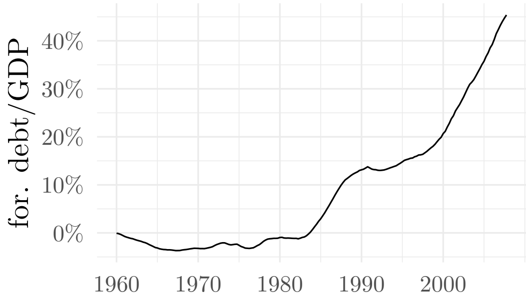
Rising inequality and KURJ was certainly not the only driver of mortgage and housing boom. In order to put the quantitative results into perspective, we use our model to simulate the effects of the major supply side mechanism—the Global Saving Glut.

The Global Saving Glut refers to the accumulation of external debt, i.e. the cumulative current account deficit which is depicted in Figure 13. The cumulative current account was roughly zero in 1980, then started to rise and reached -40% of GDP in 2006. That is, the US was a net debtor with net debt amounting to 40% of GDP.[29] Bernanke (2005) proposes the steep increase in the global demand for savings—especially from China and India—as a potential explanation for this rise in foreign debt. He argues that these savings flowed into the US economy, building up the US debt position.

Through the lens of our model, the Global Saving Glut changes the market clearing condition (5) of the asset and mortgage market. Exogenous asset supply is given by a_t^S, where $a_t^S/\bar{y}_t$ is the cumulative current account from Figure 13 ($\bar{y}_t$ is average pre-tax earnings, our measure of GDP).

[29] Gourinchas et al. (2017) estimate that the precise net foreign asset position was less negative due to valuation effects.

Figure 13: US Current Account Deficit



Notes: This figure shows the cumulative current account deficit (which is approximately US external debt) as a fraction of GDP. Source: BEA and FRED. For details, see Appendix B.

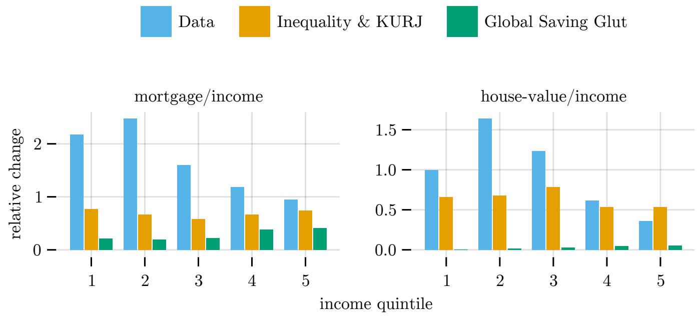
Figure 14: Steady State Effects for the Global Saving Glut

Notes: This figure shows relative changes in aggregate variables between the steady states in 1980 and 2007 for different scenarios and the corresponding changes in the data. Data: DINA. *Saving Glut*: Constant inequality and reference measure $\bar{h}$, varying a^S to match net foreign debt position (see Figure 13). Data: DINA.

Figure 14 shows that the Global Saving Glut indeed causes a substantial increase in the mortgage-to-income ratio that is of the same order of magnitude. In contrast to the combination of inequality and KURJ, however, the Global Saving Glut can only account for a weak increase in house prices if inequality is held fixed at the 1980 level. If we also change inequality to the level in 2007, the combination of rising income inequality and the Global Saving Glut generates a moderate house price increase by 22%. The increase in the mortgage-to-income ratio does not change significantly when switching on the change in income inequality (37% instead of 32%).

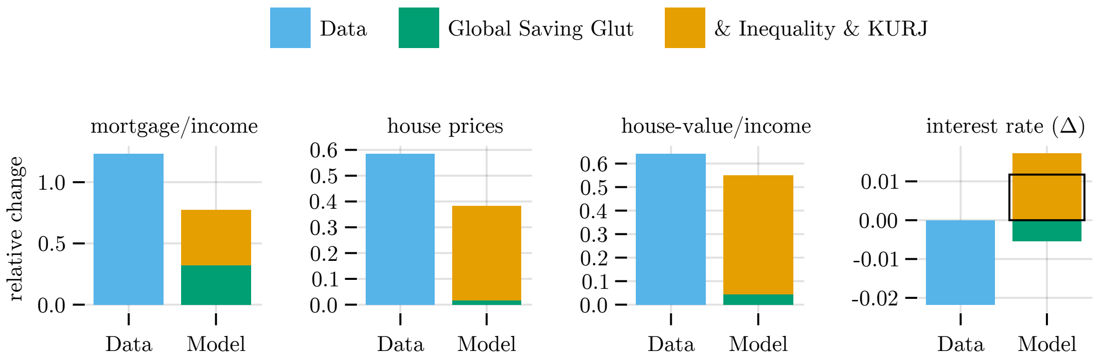
Figure 15 shows the change in mortgages and house-values relative to income across the income distribution for the Global Saving Glut experiment. We find that the mortgage boom induced by the Global Saving Glut is mainly concentrated at the top of the income distribution. Independent of the average size of the effect, the same holds for house values.

Figure 15: Heterogeneity Across the Income Distribution



Notes: This figure shows the relative changes in the mortgage-to-income ratio, the house-value-to-income ratio and leverage (mortgage-to-house-value ratio) across the income distribution. Data: DINA.

Figure 16: Decomposition of the three mechanisms



Notes: This figure shows the joint effect of the Global Saving Glut and the combination of rising inequality and KURJ. The blue bars correspond to the effects of the Global Saving Glut and the green bars show the marginal effect adding rising inequality and KURJ. The sum of the blue and green bars gives the total effect. Data: DINA.

6.2.1 Combining Inequality and KURJ with the Global Saving Glut

The results so far suggest that these demand- and supply-side mechanisms complement each other quite well. Rising inequality and KURJ generate a debt boom across the income distribution and drive up house prices and house values relative to income. The Global Saving Glut also contributes to the debt boom and puts downward pressure on the real interest rate. In the final part of the quantitative analysis, we therefore combine rising inequality and KURJ with the Global Saving Glut.

Figure 16 shows the joint effect of both mechanisms combined as well as the contribution of each mechanism. In particular, we first solve the model with only the Global Saving Glut (blue bar) and then add rising inequality and KURJ and re-solve the model to get the joint effect.

Both mechanisms together generate an increase in the mortgage-to-income ratio of 77%, an increase in house prices of 38% and an increase in the house-value-to-income ratio of 55%. We further find that the Global Saving Glut contributes slightly less to the increase in mortgages than the combination of rising inequality and KURJ. We further find that the contributions of the Global Saving Glut and the combination of rising inequality and KURJ are of the same order of magnitude. However, virtually all of the increase in house prices and house values relative to income can be attributed to rising inequality and KURJ. The total effect on the interest rate is positive even though the Global Saving Glut pulls it down.

7 Conclusion

This analysis was motivated by the parallel increase in top income inequality and mortgage debt in the US between 1980 and 2007. We first document novel aspects of the mortgage and housing boom. Using state-year variation, we find a strong positive relationship between lagged top incomes and (non-rich) household debt. Importantly, this state-level relationship is entirely driven by mortgage debt suggesting that housing played an important role in the transmission of rising top income inequality to rising household debt. Our finding that rising top incomes also drive up house prices underscores this and implies that rising housing demand contributed to the increase in mortgage debt.

Attempting to rationalize the sharp increase in household debt, previous studies have focused on supply-side mechanisms and the role of falling interest rates. Motivated by our empirical findings, we investigate—analytically and quantitatively—a demand-side mechanism to complement existing supply-side theories and rationalize the state-level findings. Our model, where households care not only about own consumption and housing but also about the housing benchmark set by the rich, is consistent with the findings of our empirical analysis in Section 2. The model predicts that *mortgage debt of the non-rich* and *house prices* rise as top incomes rise, while *non-mortage debt of the non-rich* is not affected by changes in top incomes (see Sections 4.3 and 4.4).

While the mechanism is consistent with the evidence along many dimensions, it generates a counter-factual prediction regarding interest rates. While the Global Saving Glut can rationalize falling interest rates, we find that it cannot rationalize the shift towards housing, particularly in the bottom half of the income distribution. We emphasize that we see rising inequality and KURJ as an important complement to supply-side drivers of the household debt boom. Our analysis suggests that the combination of supply- and demand-side factors is important in order to paint a complete picture of the US debt boom.

References

Abel, A. B. (1990): "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80, 38–42.

Achdou, Y., J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll (2021): "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," *Review of Economic Studies*, rdab002.

Adam, K., P. Kuang, and A. Marcet (2012): "House Price Booms and the Current Account," *NBER Macroeconomics Annual*, 26, 77–122.

Alvaredo, F., A. B. Atkinson, T. Piketty, E. Saez, and G. Zucman (2016): "The World Wealth and Income Database," .

Auclert, A. and M. Rognlie (2018): "Inequality and aggregate demand," Tech. rep., National Bureau of Economic Research.

Autor, D. H., L. F. Katz, and M. S. Kearney (2008): "Trends in Us Wage Inequality: Revising the Revisionists," *The Review of Economics and Statistics*, 90, 300–323.

Badarinza, C. (2019): "Mortgage debt and social externalities," *Review of Economic Dynamics*, 34, 43–60.

Bajari, P., P. Chan, D. Krueger, and D. Miller (2013): "A dynamic model of housing demand: Estimation and policy implications," *International Economic Review*, 54, 409–442.

Ballester, C., A. Calvó-Armengol, and Y. Zenou (2006): "Who's Who in Networks. Wanted: The Key Player," *Econometrica*, 74, 1403–1417.

Bellet, C. (2019): "The McMansion Effect: Top Size Inequality, House Satisfaction and Home Improvement in US Suburbs," Working paper, Erasmus University Rotterdam.

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra (2018): "House Prices and Consumer Spending," *Review of Economic Studies*, 85, 1502–1542.

Bernanke, B. S. (2005): "The Global Saving Glut and the U.S. Current Account Deficit," Speech 77, Board of Governors of the Federal Reserve System (U.S.).

Bertrand, M. and A. Morse (2016): "Trickle-Down Consumption," *Review of Economics and Statistics*, 98, 863–879.

Bursztyn, L., F. Ederer, B. Ferman, and N. Yuchtman (2014): "Understanding mechanisms underlying peer effects: Evidence from a field experiment on financial decisions," *Econometrica*, 82, 1273–1301.

Bursztyn, L., B. Ferman, S. Fiorin, M. Kanz, and G. Rao (2017): "Status Goods: Experimental Evidence from Platinum Credit Cards," *The Quarterly Journal of Economics*, qjx048.

Campbell, J. Y. and J. H. Cochrane (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.

Card, D., A. Mas, E. Moretti, and E. Saez (2012): "Inequality at Work: The Effect of Peer Salaries on Job Satisfaction," *American Economic Review*, 102, 2981–3003.

Carr, M. D. and E. E. Wiemers (2016): "Recent Trends in the Variability of Men's Earnings: Evidence from Administrative and Survey Data," .

——— (2018): "New evidence on earnings volatility in survey and administrative data," in *AEA Papers and Proceedings*, vol. 108, 287–91.

Charles, K. K., E. Hurst, and N. Roussanov (2009): "Conspicuous Consumption and Race*," *The Quarterly Journal of Economics*, 124, 425–467.

Clark, A. E. and C. Senik (2010): "Who Compares to Whom? The Anatomy of Income Comparisons in Europe," *Economic Journal*, 120, 573–594.

Coibion, O., Y. Gorodnichenko, M. Kudlyak, and J. Mondragon (2020): "Greater Inequality and Household Borrowing: New Evidence from Household Data," *Journal of the European Economic Association*, 18, 2922–2971.

De Giorgi, G., A. Frederiksen, and L. Pistaferri (2020): "Consumption network effects," *Review of Economic Studies*, 87, 130–163.

Duesenberry, J. (1949): *Income, Saving and the Theory of Consumer Behavior*, Cambridge, MA, USA: Harvard University Press.

Favilukis, J., S. C. Ludvigson, and S. van Nieuwerburgh (2017): "The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium," *Journal of Political Economy*, 125, 140–223.

Ferrer-i-Carbonell, A. (2005): "Income and Well-being: An Empirical Analysis of the Comparison Income Effect," *Journal of Public Economics*, 89, 997–1019.

Flavin, M. and S. Nakagawa (2008): "A model of housing in the presence of adjustment costs: A structural interpretation of habit persistence," *American Economic Review*, 98, 474–95.

Fogli, A. and V. Guerrieri (2019): "The End of the American Dream? Inequality and Segregation in US Cities," Working Paper 26143, National Bureau of Economic Research.

Frank, R. (2013): *Falling behind: How Rising Inequality Harms the Middle Class*, vol. 4, Univ of California Press.

Gali, J. (1994): "Keeping up with the Joneses: Consumption externalities, portfolio choice, and asset prices," *Journal of Money, Credit and Banking*, 26, 1–8.

Garriga, C. and A. Hedlund (2020): "Mortgage Debt, Consumption, and Illiquid Housing Markets in the Great Recession," *American Economic Review*, 110, 1603–34.

Garriga, C., R. Manuelli, and A. Peralta-Alva (2019): "A macroeconomic model of price swings in the housing market," *American Economic Review*, 109, 2036–72.

Ghiglino, C. and S. Goyal (2010): "Keeping up with the Neighbors: Social Interaction in a Market Economy," *Journal of the European Economic Association*, 8, 90–119.

Gourinchas, P.-O., H. Rey, and N. Govillot (2017): "Exorbitant privilege and exorbitant duty," Working paper, University of California, Berkeley.

Guvenen, F., G. Kaplan, J. Song, and J. Weidner (2018): "Lifetime incomes in the United States over six decades," .

Guvenen, F., F. Karahan, S. Ozkan, and J. Song (2021): "What Do Data on Millions of U.S. Workers Reveal About Life-Cycle Earnings Dynamics?" *Econometrica.*

Guvenen, F., S. Ozkan, and J. Song (2014): "The Nature of Countercyclical Income Risk," *Journal of Political Economy*, 122, 621–660.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2020): "The Slope of the Phillips Curve: Evidence from U.S. States," Working Paper 28005, National Bureau of Economic Research.

Heathcote, J., K. Storesletten, and G. L. Violante (2010): "The Macroeconomic Implications of Rising Wage Inequality in the United States," *Journal of Political Economy*, 118, 681–722.

——— (2017): "Optimal tax progressivity: An analytical framework," *Quarterly Journal of Economics*, 132, 1693–1754.

Jordà, Ò., M. Schularick, and A. M. Taylor (2016): "The Great Mortgaging: Housing Finance, Crises and Business Cycles," *Economic Policy*, 31, 107–152.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2014): "The effects of the saving and banking glut on the US economy," *Journal of International Economics*, 92, S52–S67.

——— (2019): "Credit Supply and the Housing Boom," *Journal of Political Economy*, 127, 1317–1350.

Kaplan, G., K. Mitman, and G. L. Violante (2020): "The housing boom and bust: Model meets evidence," *Journal of Poltical Economy.*

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary policy according to HANK," *American Economic Review*, 108, 697–743.

Katz, L. F. and K. M. Murphy (1992): "Changes in Relative Wages, 1963–1987: Supply and Demand Factors," *The Quarterly Journal of Economics*, 107, 35–78.

Kopczuk, W., E. Saez, and J. Song (2010): "Earnings Inequality and Mobility in the United States: Evidence from Social Security Data since 1937," *The Quarterly Journal of Economics*, 125, 91–128.

Kuhn, M., M. Schularick, and U. I. Steins (2017): "The Great American Debt Boom, 1949–2013," Working paper, University of Bonn.

Kuhn, P., P. Kooreman, A. Soetevent, and A. Kapteyn (2011): "The effects of lottery prizes on winners and their neighbors: Evidence from the Dutch postcode lottery," *The American Economic Review*, 101, 2226–2247.

Kumhof, M., R. Rancière, and P. Winant (2015): "Inequality, Leverage, and Crises," *American Economic Review*, 105, 1217–45.

Landvoigt, T. (2017): "Housing demand during the boom: The role of expectations and credit constraints," *Review of Financial Studies*, 30, 1865–1902.

Ljungqvist, L. and H. Uhlig (2000): "Tax policy and aggregate demand management under catching up with the Joneses," *American Economic Review*, 356–366.

Luttmer, E. F. P. (2005): "Neighbors as Negatives: Relative Earnings and Well-Being," *Quarterly Journal of Economics*, 120, 963–1002.

Mian, A., L. Straub, and A. Sufi (2021): "Indebted Demand," *The Quarterly Journal of Economics*, qjab007.

Mian, A. R., L. Straub, and A. Sufi (2020): "The Saving Glut of the Rich and the Rise in Household Debt," Working Paper 26941, National Bureau of Economic Research.

Ogaki, M. and C. M. Reinhart (1998): "Measuring intertemporal substitution: The role of durable goods," *Journal of Political Economy*, 106, 1078–1098.

Perez-Truglia, R. (2019): "The Effects of Income Transparency on Well-Being: Evidence from a Natural Experiment," Working Paper 25622, National Bureau of Economic Research.

Piazzesi, M. and M. Schneider (2016): "Chapter 19 - Housing and Macroeconomics ," Elsevier, vol. 2 of *Handbook of Macroeconomics*, 1547 – 1640.

Piazzesi, M., M. Schneider, and S. Tuzel (2007): "Housing, Consumption and Asset Pricing," *Journal of Financial Economics*, 83, 531 – 569.

Piketty, T., E. Saez, and G. Zucman (2018): "Distributional national accounts: methods and estimates for the United States," *Quarterly Journal of Economics*, 133, 553–609.

Rajan, R. (2010): *Fault Lines: How Hidden Fractures Still Threaten the World Economy*, Princeton, NJ, USA: Princeton University Press.

Saiz, A. (2010): "The geographic determinants of housing supply," *Quarterly Journal of Economics*, 125, 1253–1296.

Smith, M., D. Yagan, O. Zidar, and E. Zwick (2019): "Capitalists in the Twenty-first Century," *The Quarterly Journal of Economics*, 134, 1675–1745.

Solnick, S. and D. Hemenway (2005): "Are Positional Concerns Stronger in Some Domains Than in Others?" *American Economic Review*, 95, 147–151.

Stiglitz, J. (2009): "The global crisis, social protection and jobs," *International Labour Review*, 148, 1–13.

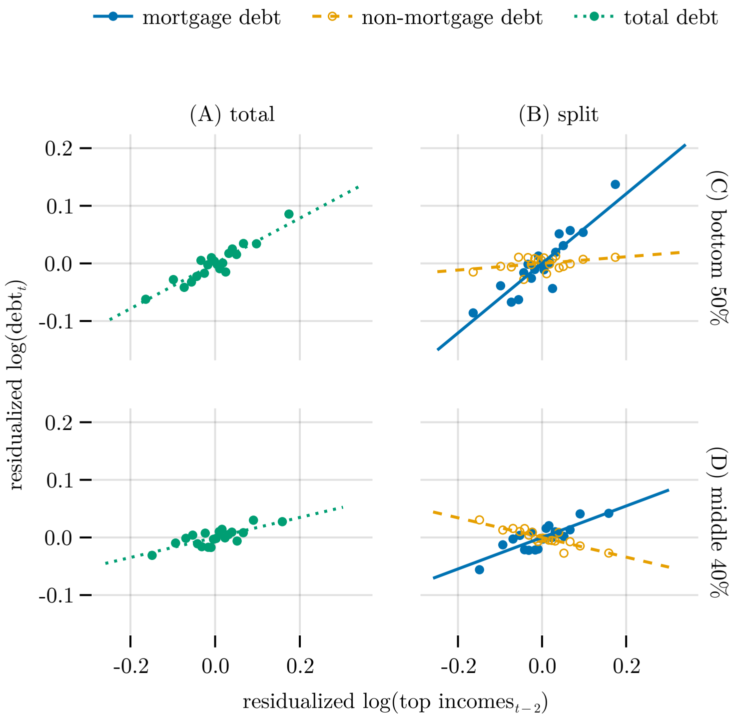
Straub, L. (2018): "Consumption, Savings, and the Distribution of Permanent Income," *Unpublished manuscript, Harvard University.*

van Treeck, T. (2014): "Did Inequality Cause the U.S. Financial Crisis?" *Journal of Economic Surveys*, 28, 421–448.

Veblen, T. (1899): *The Theory of the Leisure Class*, Macmillan: George Allen and Unwin.

A Additional Figures and Tables for Empirical Analysis

Figure 17: Residualized Household Debt and Lagged Top Incomes by Income Groups



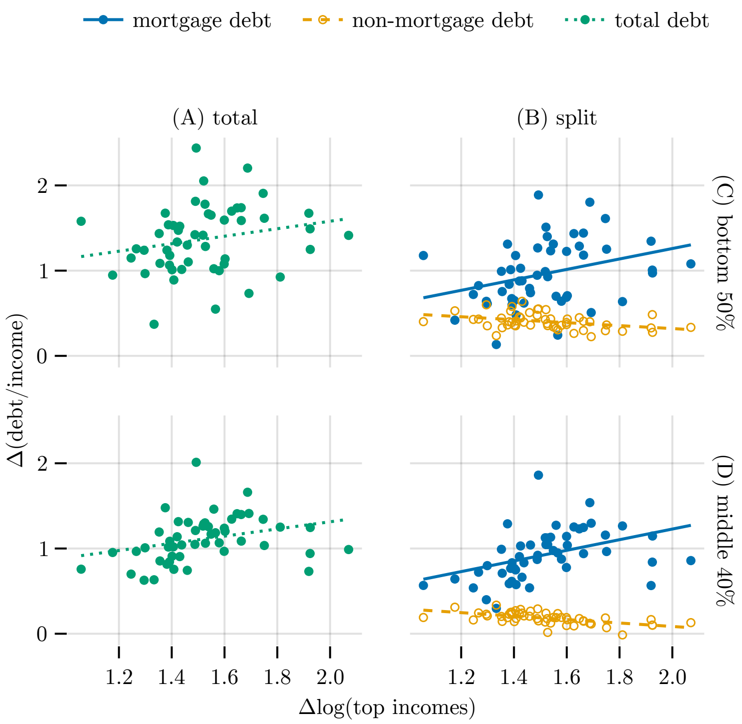
Notes: This figure shows the relationship between log debt (total, mortgage, non-mortgage) of households in the middle 40 and bottom 50 percent of the income distribution and the second lag of log average top 10% incomes conditional on state and year fixed effects and non-rich income. All variables are residualized using state and year fixed effects. The slope of the regression line is the OLS estimate of β reported in Table 5. The figure shows averages in 20 equally sized bins of the x-variable.

Table 5: Top Incomes and Household Debt: Fixed Effects Regressions by Income Groups

	log(debt$_{s,t}$)					
	(A) total		(B) mortgage		(C) non-mortgage	
	bottom 50%	middle 40%	bottom 50%	middle 40%	bottom 50%	middle 40%
	(1)	(2)	(3)	(4)	(5)	(6)
log(top incomes$_{s,t-2}$)	0.391***	0.148**	0.567***	0.211**	0.099	-0.115
	(0.112)	(0.068)	(0.163)	(0.098)	(0.069)	(0.084)
log(own income$_{s,t}$)	0.429***	0.949***	0.484***	1.122***	0.365***	0.560***
	(0.044)	(0.082)	(0.065)	(0.107)	(0.027)	(0.092)
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	OLS	OLS	OLS
N	1,224	1,224	1,224	1,224	1,224	1,224
R^2	0.950	0.976	0.895	0.963	0.970	0.972

Notes: This table shows the estimation results corresponding to equation 1. The dependent variable is either total, mortgage or non-mortgage debt in the middle 40 and bottom 50 percent of the income distribution. Robust standard errors in parentheses. The stars indicate the range of the p value: $^{***} \leq 0.01 \leq {}^{**} \leq 0.05 \leq {}^{*} \leq 0.1$.

Figure 18: Long-Run Changes in Household Debt and Top Incomes by Income Groups



Notes: This figure shows the relationship between the change in debt-to-income (total, mortgage, non-mortgage) of households in the middle 40 and bottom 50 percent of the income distribution and the change in the log of average top 10% incomes between 1982 and 2007 across US states.

Table 6: Top Incomes and Household Debt: Long Run Changes

	Δlog(debt/income)											
	(A) total				(B) mortgage				(C) non-mortgage			
	population	bottom 90%	bottom 50%	middle 40%	population	bottom 90%	bottom 50%	middle 40%	population	bottom 90%	bottom 50%	middle
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	
Δlog(top incomes)	-0.101	0.412**	0.440	0.421**	0.124	0.618***	0.613**	0.624***	-0.225***	-0.206***	-0.173***	-0.2
	(0.128)	(0.184)	(0.279)	(0.185)	(0.128)	(0.187)	(0.258)	(0.193)	(0.025)	(0.038)	(0.063)	(0
Estimator	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	
N	51	51	51	51	51	51	51	51	51	51	51	
R^2	0.013	0.093	0.048	0.096	0.019	0.182	0.103	0.176	0.633	0.376	0.135	

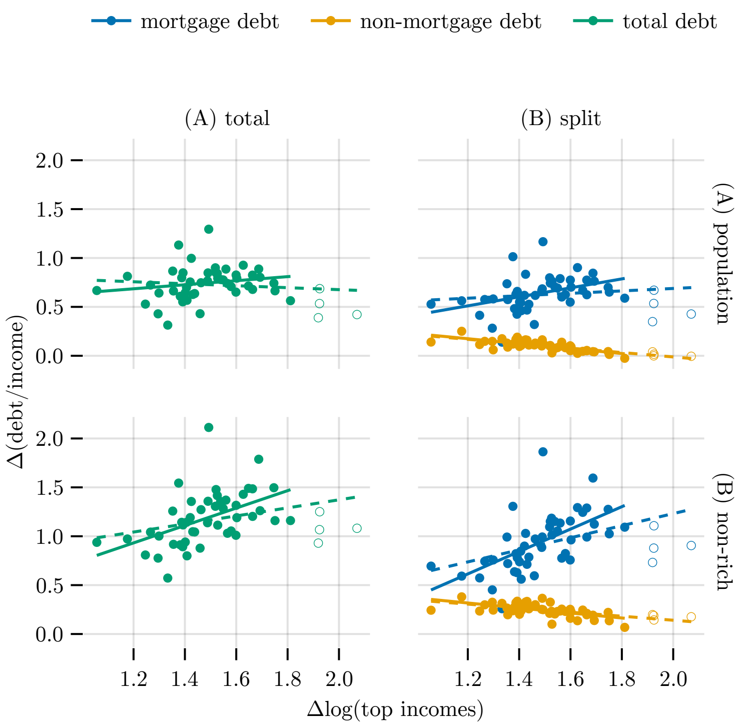
Notes: This table shows the results of OLS regressions of the long-run change in the debt-to-income ratio (total, mortgage, non-mortgage) of different groups (all households, bottom 90%, middle 40%, bottom 50%) on the long-run change in the log of average top 10% incomes.

Table 7: Top Incomes and Household Debt: Long Run Changes *Without Outliers*

	Δlog(debt/income)					
	(A) total		(B) mortgage		(C) non-mortgage	
	population	non-rich	population	non-rich	population	non-rich
	(1)	(2)	(3)	(4)	(5)	(6)
Δlog(top incomes)	0.207	0.891***	0.460***	1.147***	-0.253***	-0.256***
	(0.161)	(0.230)	(0.158)	(0.228)	(0.033)	(0.051)
Estimator	OLS	OLS	OLS	OLS	OLS	OLS
N	47	47	47	47	47	47
R^2	0.036	0.250	0.159	0.359	0.561	0.357

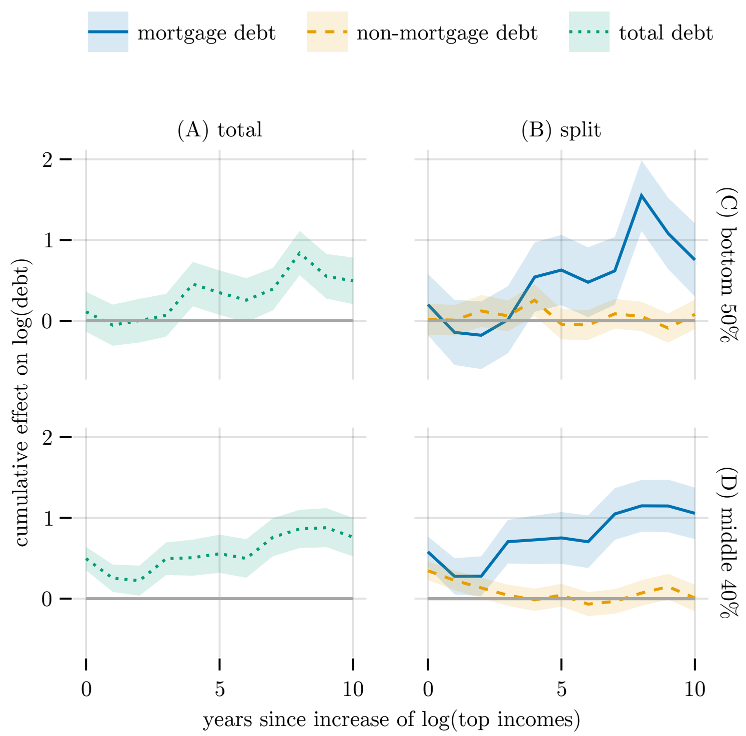
Notes: This table shows the results of OLS regressions of the long-run change in the debt-to-income ratio (total, mortgage, non-mortgage) of all households and non-rich households on the long-run change in the log of average top 10% incomes using all states except for the following 4 states with especially high growth in top incomes: New York, Massachusetts, Connecticut and the District of Columbia.

Figure 19: Long-Run Changes in Household Debt and Top Incomes by Income Groups *Without Outliers*



Notes: This figure shows the relationship between the change in debt-to-income (total, mortgage, non-mortgage) of all households and non-rich households and the change in the log of average top 10% incomes between 1982 and 2007 across all US states except for the following 4 states with especially high growth in top incomes: New York, Massachusetts, Connecticut and the District of Columbia. These states are depicted using hollow markers.

Figure 20: Dynamic Effects of Top Incomes on Household Debt by Income Groups



Notes: This figure shows the cumulative effect of a 1% change in top 10% incomes on total, mortgage and non-mortgage debt of different income groups estimated from equation 2. The confidence bands are constructed using a significance level of 5%.

B Data Sources

Figure 1: Aggregate debt and inequality We use data on outstanding household debt from the US Flow of Funds, retrieved from FRED: total debt (`TLBSHNO`) and mortgages (`HMLBSHNO`). *Other debt* is constructed as the difference between total debt and mortgages. Debt is displayed as a share of nominal GDP (Bureau of Economic Analysis, BEA, via FRED: `GDP`).

The top 10% income share is from the World Wealth and Income Database (Alvaredo et al., 2016).

Figure 13: Net foreign debt position of the US We use the current account and GDP series from the BEA, retrieved via FRED (`BOPBCA`, `GDP`). Following Gourinchas et al. (2017) we compute the cumulative sum of the current account

$$\text{cum CA}_t = \sum_{i=1960}^{t} \text{CA}_t$$

and show it as a fraction of GDP in that given year $\frac{\text{cum CA}_t}{\text{GDP}_t}$.

C Proofs

C.1 Lemmas

Lemma 1. *The necessary conditions for an optimum of the households' problem are*

$$u_c\big(c_t, s(h_t, \bar{h}_t)\big) = \lambda_t \tag{7}$$

$$u_s\big(c_t, s(h_t, \bar{h}_t)\big) s_h(h_t, \bar{h}_t) = \lambda_t(r+\delta)p \tag{8}$$

$$\dot{\lambda}_t - \rho\lambda_t = -r\lambda_t \tag{9}$$

where λ is the co-state in the continuous time optimization problem.

Proof. Without adjustment costs, the two endogenous state variables a_t and h_t collapse into one state variable net worth w_t.

$$\dot{w}_t = rw_t + y_t - (r+\delta)ph_t - c_t$$

The present-value Hamiltonian is

$$H(w, h, c, \lambda) = u\big(c, s(h, \bar{h})\big) + \lambda\big(rw_t + y_t - (r+\delta)ph_t - c_t\big),$$

where w is the state, c and h are the controls and λ is the co-state. The necessary conditions are

$$\frac{\partial H(w_t, h_t, c_t, \lambda_t)}{\partial c} = u_c\big(c_t, s(h_t, \bar{h}_t)\big) - \lambda_t = 0$$

$$\frac{\partial H(w_t, h_t, c_t, \lambda_t)}{\partial h} = u_s\big(c_t, s(h_t, \bar{h}_t)\big) s_h(h_t, \bar{h}_t) - \lambda_t(r+\delta)p = 0$$

$$\dot{\lambda}_t - \rho\lambda_t = \frac{\partial H(w_t, h_t, c_t, \lambda_t)}{\partial w} = -r\lambda_t. \qquad \square$$

Lemma 2. *Under our assumption of CRRA-CES preferences, the optimal relation of c_t and h_t is given by*

$$\frac{\xi}{1-\xi}\left(\frac{s(h_t, \bar{h}_t)}{c_t}\right)^{\varepsilon-1} s_h(h_t, \bar{h}_t) = (r+\delta)p. \tag{10}$$

Further assuming Assumption 3 yields

$$c_t = \kappa_0 h_t - \kappa_0 \phi \bar{h}_t, \quad \textit{where } \kappa_0 = \left((r+\delta)p\frac{1-\xi}{\xi}\right)^{\frac{1}{1-\varepsilon}}. \tag{11}$$

Proof. Combining conditions (7) and (8) yields

$$\frac{u_s(c_t, s_t)}{u_c(c_t, s_t)} s_h(h_t, \bar{h}_t) \stackrel{!}{=} (r+\delta)p.$$

For the given CRRA-CES preferences the marginal utilites are given by

$$u_c(c_t, s_t) = ((1-\xi)c_t^{\varepsilon} + \xi s_t^{\varepsilon})^{\frac{1-\gamma}{\varepsilon}-1}(1-\xi)c_t^{\varepsilon-1}$$

$$u_s(c_t, s_t) = ((1-\xi)c_t^{\varepsilon} + \xi s_t^{\varepsilon})^{\frac{1-\gamma}{\varepsilon}-1}\xi s_t^{\varepsilon-1}. \tag{12}$$

Thus,

$$\frac{u_s(c_t, s_t)}{u_c(c_t, s_t)} = \frac{\xi}{1-\xi}\left(\frac{s_t}{c_t}\right)^{\varepsilon-1}.$$

Plugging in above yields the first statement. Using Assumption 3 we get

$$\frac{\xi}{1-\xi}\Big(\frac{h_t - \phi\bar{h}}{c_t}\Big)^{\varepsilon-1} = (r+\delta)p.$$
$$\Big(\frac{c_t}{h_t - \phi\bar{h}}\Big) = \Big((r+\delta)p\frac{1-\xi}{\xi}\Big)^{\frac{1}{1-\varepsilon}} = \kappa_0$$
$$c_t = \kappa_0 h_t - \kappa_0\phi\bar{h}_t$$ □

Lemma 3. *Under the assumption of time-constant house prices p, and all previous assumptions of this section, individual choices c_t, h_t are constant over time.*

Proof. The costate λ is constant over time. This follows from using Assumption 1 in condition (9), which gives $\dot{\lambda}_t = 0$.

Plugging in (11) in condition (8) one gets that an decreasing function of h is constant over time, thus h_t is constant over time. Knowing that h_t constant over time, and a similar argument for condition (7) it follows that c_t is constant over time. □

C.2 Proof of Proposition 1

From the lemmas above we get that

$$c = \kappa_0 s(h,\bar{h}) = \kappa_0 h - \kappa_0\phi\bar{h}.$$

Using the lifetime budget constraint we get

$$\begin{aligned}
\mathcal{Y} &:= ra_0 + y = ph(r+\delta) + c \\
&= h\Big(p(r+\delta)+\kappa_0\Big) - \kappa_0\phi\bar{h} \\
\implies h &= \frac{\mathcal{Y} + \kappa_0\phi\bar{h}}{p(r+\delta)+\kappa_0} = \underbrace{\frac{1}{p(r+\delta)+\kappa_0}}_{\kappa_2}\mathcal{Y} + \underbrace{\frac{\kappa_0}{p(r+\delta)+\kappa_0}}_{\kappa_1}\phi\bar{h} = \kappa_2\mathcal{Y} + \kappa_1\phi\bar{h}
\end{aligned} \tag{13}$$

where

$$\kappa_1 := \frac{\kappa_0}{p(r+\delta)+\kappa_0} = \frac{1}{\frac{p(r+\delta)}{\kappa_0}+1} \in (0,1)$$

since

$$\frac{p(r+\delta)}{\kappa_0} = \Big(\frac{1}{(r+\delta)p}\Big)^{\frac{1}{1-\varepsilon}-1}\Big(\frac{\xi}{1-\xi}\Big)^{\frac{1}{1-\varepsilon}} > 0.$$

Stacking equations (13) for and using $\bar{\boldsymbol{h}} = G\boldsymbol{h}$

$$\boldsymbol{h} = \kappa_2\boldsymbol{\mathcal{Y}} + \kappa_1\phi G\boldsymbol{h}$$
$$\boldsymbol{h} = (I - \kappa_1\phi G)^{-1}\kappa_2\boldsymbol{\mathcal{Y}} = \Big(\sum_{i=0}^{\infty}(\kappa_1\phi G)^i\Big)\kappa_2\boldsymbol{\mathcal{Y}}.$$

Moreover,

$$
\begin{aligned}
\bar{\boldsymbol{h}} = G\boldsymbol{h} &= \frac{\kappa_1\phi}{\kappa_1\phi} G\Big(\sum_{i=0}^{\infty}(\kappa_1\phi G)^i\Big)\kappa_2\boldsymbol{\mathcal{Y}} \\
&= \frac{1}{\kappa_1\phi}\Big(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\Big)\kappa_2\boldsymbol{\mathcal{Y}} \\
&= \frac{1}{\kappa_0\phi}\Big(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\Big)\boldsymbol{\mathcal{Y}}
\end{aligned}
$$

$(I-\kappa_1\phi G)^{-1}$ is a Leontief inverse. It exists if the matrix power series $\sum_{i=0}^{\infty}(\kappa_1\phi G)^i$ converges[30]. In that case

$$(I-\kappa_1\phi G)^{-1} = \sum_{i=0}^{\infty}(\kappa_1\phi G)^i.$$

Now, we calculate debt.

$$-ra = y - \delta ph - c$$

using C.2,

$$
\begin{aligned}
&= y - \delta ph - \kappa_0 h + \kappa_0\phi\bar{h} \\
&= y - (\delta p + \kappa_0)h + \kappa_0\phi\bar{h} \\
-r\boldsymbol{a} &= \boldsymbol{y} - (\delta p + \kappa_0)\underbrace{\Big(\sum_{i=0}^{\infty}(\kappa_1\phi G)^i\Big)}_{=I+\left(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\right)}\kappa_2\boldsymbol{\mathcal{Y}} + \Big(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\Big)\boldsymbol{\mathcal{Y}} \\
&= \boldsymbol{y} - \kappa_3\boldsymbol{\mathcal{Y}} + (1-\kappa_3)\Big(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\Big)\boldsymbol{\mathcal{Y}}
\end{aligned}
$$

where

$$\kappa_3 = (\delta p + \kappa_0)\kappa_2 = \frac{\delta p + \kappa_0}{p(r+\delta)+\kappa_0} = \frac{1}{1+\frac{pr}{\delta p + \kappa_0}} \in (0,1).$$

[30]This is the case for all nilpotent matrices (there exists a power p such that $G^p = 0I$) (there are no infinitely-long paths in the network) or if all eigenvalues of $\kappa_1\phi G$ are between 0 and 1. This holds whenever G can be interpreted as a Markov Chain.

D Details on the Earnings Process

The innovations of both the transitory and persistent process are drawn from mixture distributions to match higher order moments of income risk and impulse response functions. Finally, Guvenen et al. (2021) show that a non-employment shock with z-dependent shock probabilities greatly improves the model fit.[31]

If employed, individual pre-tax earnings are given by

$$y_{it}^{\text{pot}} = \exp(\tilde{\alpha}^i + z_{it} + \epsilon_{it}).$$

We will refer to y^{pot} as *potential earnings*. The actual pre-tax earnings (taking into account unemployment) are

$$y_{it} = (1 - \nu_{it}) y_{it}^{\text{pot}},$$

where

$$\begin{aligned}
\tilde{\alpha}_i &\sim \mathcal{N}(\mu_\alpha, \sigma_\alpha), \\
\mathrm{d}z_{it} &= -\theta^z z_{it} \mathrm{d}t + \mathrm{d}J_{it}^z, \\
\mathrm{d}\epsilon_{it} &= -\theta^\epsilon z_{it} \mathrm{d}t + \mathrm{d}J_{it}^\epsilon.
\end{aligned}$$

J_{it}^z is a jump-process that arrives at rate λ^z. The size of the jump, η_{it}^z is drawn from a mixture of two normal distributions,

$$\eta_{it}^z = \begin{cases} \mathcal{N}\big(\mu^z(1-p^z), \sigma_1^z\big) & \text{with prob. } p^z \\ \mathcal{N}\big(-p^z\mu^z, \sigma_2^z\big) & \text{with prob. } 1-p^z. \end{cases}$$

Similarly, the jump process for the transitory process arrives at rate λ^ϵ and the jump size, η_{it}^z is drawn from a mixture of two normal distributions,

$$\eta_{it}^\epsilon = \begin{cases} \mathcal{N}\big(-\epsilon_{it} + \mu^\epsilon(1-p^\epsilon), \sigma_1^\epsilon\big) & \text{with prob. } p^\epsilon \\ \mathcal{N}\big(-\epsilon_{it} - p^\epsilon\mu^\epsilon, \sigma_2^\epsilon\big) & \text{with prob. } 1-p^\epsilon. \end{cases}$$

The key difference between the persistent and the transitory process is that the jumps in the former are added to the current state whereas the jumps in the latter process reset the process such that the post-jump state is centered around zero.

The nonemployment shock arrives at rate $\lambda_0^\nu(z_{it})$ and has average duration $1/\lambda_1^\nu$. Specifically, the arrival probability as a function of the current state of the persistent process is modeled as

$$\lambda_0^\nu(z_{it})\mathrm{d}t = \frac{\exp\big(a + b z_{it}\big)}{1 + \exp\big(a + b z_{it}\big)}.$$

Table 8 shows all parameters of our continuous time earnings process.

[31] The only component that is missing compared to the Benchmark process is fixed heterogeneous income profiles, i.e. ex-ante permanent heterogeneity in lifecycle income growth rates.

Table 8: Earnings Process Parameters

Parameter		Value
Fixed Effects		
μ_α	mean	$2.7408 + 0.4989\bar{t} - 0.1137\bar{t}^2$
σ_α	standard deviation	0.467
Persistent Process		
λ^z	arrival rate	1.0
θ^z	mean reversion rate	$-\log(0.983)$
p^z	mixture probability	0.267
μ^z	location parameter	-0.194
σ_1^z	std. dev. of first Normal	0.444
σ_2^z	std. dev. of second Normal	0.076
σ_0^z	std. dev. of z_{i0}	0.495
Transitory Shocks		
λ^ϵ	arrival rate	1.0
θ^ϵ	mean reversion rate	0.0
p^ϵ	mixture probability	0.092
μ^ϵ	location parameter	0.352
σ_1^ϵ	std. dev. of first Normal	0.294
σ_2^ϵ	std. dev. of second Normal	0.065
Nonemployment Shocks		
a	constant	$-3.2740 - 0.8935\bar{t}$
b	slope	$-4.5692 - 2.9203\bar{t}$
λ_1^ν	exit rate	1/0.9784

E Numerical Solution for a Stationary Equilibrium

We first describe how we discretize the complex income process, then we show how to solve the partial equilibrium using a finite difference method from Achdou et al. (2021). Finally we present the algorithm used to compute equilibrium prices and reference measure.

The model was solved using version 1.2 of the `Julia` language. For a given parameterization, 200 endogenous grid points and 2000 exogenous grid points solving for a general equilibrium takes about 30 minutes on standard laptop using just one core.

For the calibration we ran the code in parallel (using 30 nodes with 16 cores) for 12 hours on a high performance cluster.

E.1 Discretizing the Income Process

Pre-tax earnings depend on four exogenous states $\theta = (\tilde{\alpha}, z, \epsilon, \nu)$,

$$y(\theta) = (1 - \nu) \exp(\tilde{\alpha} + z + \epsilon).$$

We first discretize the two jump-drift processes z and ϵ following the procedure of Kaplan et al. (2018). We discretize them separately, creating two continuous time Markov chains and combining them. The state space of the combined continuous time Markov Chain is given by

$$\{z_1, \ldots, z_{N_z}\} \times \{\epsilon_1 \ldots \epsilon_{N_\epsilon}\}.$$

Then we add non-employment states for each state, where the transition probabilities into the non-employment state are state-dependent. The state space of the CTMC with non-employment becomes

$$\{z_1, \ldots, z_{N_z}\} \times \{\epsilon_1 \ldots \epsilon_{N_\epsilon}\} \times \{0, 1\}.$$

Finally we add the permanent component $\tilde{\alpha}$. We choose $N_\alpha = 10$ grid points, where each of those grid points represents a decile of $\tilde{\alpha}$'s distribution. Conditional on drawing $\tilde{\alpha}_i$, the other three components follow the same CTMC with $N_z \cdot N_\epsilon \cdot 2$ states. Denote the changing states by $\tilde{\theta} = (z, \epsilon, \nu)$

The transition between states $\tilde{\theta}$ is given by the intensities q_{ij}. For an agent at state $\tilde{\theta}_i$ the probability of jumping to a new state $\tilde{\theta}_j$ within the time short time period Δ is approximately given by $p_{ij}(\Delta) \approx q_{ij}\Delta$. More precisely, given the intensity matrix $Q = (q_{ij})$ where $q_{ij} \geq 0$ for $i \neq j$ and $q_{ii} = -\sum_{k \neq i} q_{ik}$, the matrix of transition probabilities is given by

$$P(\Delta) = \exp(-\Delta Q),$$

where exp is the matrix exponential. $P(\Delta)$ is a stochastic matrix.

E.2 Partial Equilibrium given p, r, $\bar{h}$

Given prices (p, r) and reference measure $\bar{h}$ the households' problem can be characterized by a coupled system of partial differential equations: the Hamilton-Jacobi-Bellman (HJB) equation and the Kolmogorov forward (KF) equation. The HJB equation describes the optimization problem of the households and the KF equation describes the evolution of the cross-sectional distribution $\mu(\mathrm{d}a, \mathrm{d}h, \mathrm{d}y)$.

We solve these two equations using the finite difference method from Achdou et al. (2021). The discretized system can be written as

$$\begin{aligned}\rho \boldsymbol{v} &= \boldsymbol{u}(\boldsymbol{v}) + A(\boldsymbol{v}; r, p, \bar{h})\boldsymbol{v} \\ \mathbf{0} &= \left(A(\boldsymbol{v}; r, p, \bar{h}) + M\right)^T \boldsymbol{g},\end{aligned}$$

where $\boldsymbol{v}$ is the discretized value function, $\boldsymbol{g}$ is the discretized cross-sectional distribution, $\boldsymbol{u}(\boldsymbol{v})$ is the discretized flow utility, $A(\boldsymbol{v}; r, p, \bar{h})$ is the discretized infinitesimal generator of the HJB equation (a very sparse matrix) and M is a matrix that corrects the intensities for births and deaths. The discretized system reveals how tightly coupled the HJB and KF equations are. The matrix $A(\boldsymbol{v}; r, p, \bar{h})$ shows up in both equation. Once it is known from the solution of the HJB equation, it can be directly used to get the distribution $\boldsymbol{g}$ from the KF equation.

E.2.1 Solving the Hamilton-Jacobi-Bellman equation

We assume that housing h can be adjusted frictionlessly. So the two states h and a collapse into one, "net worth"

$$w_t = a_t + ph_t,$$

with its law of motion

$$\dot{w}_t = rw_t + y_t - (r+\delta)ph_t - c_t.$$

The collateral constraint can be rewritten in terms of w

$$\begin{aligned} w_t = ph_t + a_t &\geq ph_t - \omega ph_t \\ \implies ph_t &\leq \frac{w_t}{1-\omega}. \end{aligned}$$

The households' HJB equation is

$$\begin{aligned}(\rho + m)v(w, \theta_i) = \max_{c, h \leq \frac{w}{1-\omega}} \; & u(c, s(h, \bar{h})) \\ & + v_w(w, \theta_i)(rw + \theta_i - (r+\delta)ph - c) \\ & + \sum_{k \neq i} q_{ik}(v(w, \theta_k) - v(w, \theta_i)).\end{aligned}$$

The intensities q_{ij} are the intensities of the continuous time Markov chain from Section E.1. In order to solve this equation, we need to replace the maximum operator with the maximized Hamiltonian. That is, we need to plug in the optimal policy functions $c^*(w, y)$, $h^*(w, y)$ which are given in Corollary 4 below. The result depends on the following lemma.

Lemma 4. *When the collateral constraint is slack, we get the optimality conditions*

$$\begin{aligned} h(w, y) &= \left(\frac{1}{\tau_2}\left(\bar{h}^\phi(\rho+\delta)pv_w(w, y)\right)\right)^{-\frac{1}{\gamma}} \bar{h}^\phi \\ c(w, y) &= s(h(w, y), \bar{h})\tau_1, \end{aligned}$$

where $\tau_1 = \left((r+\delta)p\frac{1-\xi}{\xi}\bar{h}^\phi\right)^{\frac{1}{1-\varepsilon}}$ *and* $\tau_2 = ((1-\xi)\tau_1^\varepsilon + \xi)^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\xi.$

Proof. Using the optimality conditions (10) and (8) with (12) we get

$$(r+\delta)p = \frac{u_s(c,s)}{u_c(c,s)} s_h(h,\bar{h}) = \frac{\xi}{1-\xi}\Big(\frac{s(h,\bar{h})}{c}\Big)^{\varepsilon-1} s_h(h,\bar{h}) \tag{25}$$

$$(\rho+\delta)pv_w(w,y) = u_s(c,s)s_h = ((1-\xi)c^\varepsilon + \xi s^\varepsilon)^{\frac{1-\gamma}{\varepsilon}-1}\xi s^{\varepsilon-1}s_h. \tag{26}$$

Using (25) we express optimal c as a function of optimal s

$$c(h,\bar{h}) = s(h,\bar{h})\Big((r+\delta)p\frac{1-\xi}{\xi}\frac{1}{s_h(h,\bar{h})}\Big)^{\frac{1}{1-\varepsilon}}$$

using the ratio specification for s

$$= s(h,\bar{h})\Big((r+\delta)p\frac{1-\xi}{\xi}\bar{h}^\phi\Big)^{\frac{1}{1-\varepsilon}} =: s(h,\bar{h})\tau_1.$$

Then we can plug this expression into (26) and get

$$\begin{aligned}(\rho+\delta)pv_w(w,y) &= ((1-\xi)(\tau_1 s)^\varepsilon + \xi s^\varepsilon)^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\xi s^{\varepsilon-1}s_h \\ &= \underbrace{((1-\xi)\tau_1^\varepsilon + \xi)^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\xi}_{=:\tau_2} s^{1-\gamma-\varepsilon}s^{\varepsilon-1}s_h \\ &= \tau_2 s^{-\gamma}s_h\end{aligned}$$

Thus we get

$$s(h,\bar{h}) = \Big(\frac{(\rho+\delta)pv_w(w,y)}{\tau_2 s_h}\Big)^{-\frac{1}{\gamma}},$$

and using ratio-specification for s,

$$h = \Big(\frac{1}{\tau_2}\big((\rho+\delta)pv_w(w,y)\bar{h}^\phi\big)\Big)^{-\frac{1}{\gamma}}\bar{h}^\phi. \qquad \square$$

Corollary 4. *The optimal policies are given by*

$$h^*(w,y) = \begin{cases} h(w,y) & \textit{if } h(w,y) < \frac{w}{p(1-\omega)} \\ \frac{w}{p(1-\omega)} & \textit{otherwise}\end{cases}, \quad c^*(w,y) = \begin{cases} c(w,y) & \textit{if } h(w,y) < \frac{w}{p(1-\omega)} \\ \tilde{c}(w,y) & \textit{otherwise}\end{cases}$$

where $h(w,y)$ and $c(w,y)$ are from Lemma 4 and $\tilde{c}(w,y)$ is the solution to the optimality condtion for c, given $h = \frac{w}{p(1-\omega)}$,

$$v_w(w,y) = ((1-\xi)c^\varepsilon + \xi s^\varepsilon)^{\frac{1-\gamma-\varepsilon}{\varepsilon}}(1-\xi)c^{\varepsilon-1},$$

which is solved numerically.

Given the optimal policies, it is straight-forward to solve the HJB using the implicit upwind scheme in Achdou et al. (2021).

E.2.2 Solving the Kolmogorov forward equation

We construct the birth and death matrix M as in Kaplan et al. (2018) and solve for the distribution using the implicit scheme from Achdou et al. (2021).

E.3 General equilibrium: Solving for r, p and $\bar{h}$

We use the following algorithm to compute general equilibria.

0. Guess r_0, p_0 and $\bar{h}_0$
1. Clear housing markets given r_{n-1} and $\bar{h}_{n-1}$
    (a) Use Newton steps until the sign of the excess demand for housing changes
    (b) Use Bisection to find the market clearing price p_n
2. Compute the excess demand on the asset market
3. Use a Newton step to update the interest rate r_n
4. Compute the implied reference measure $\bar{h}_x$ and update $\bar{h}_n = \bar{h}_{n-1} + a(\bar{h}_x - \bar{h}_{n-1})$
5. If $r_n \approx r_{n-1}$ and $\bar{h}_n \approx \bar{h}_{n-1}$, an equilibrium has been found. If not, go back to step 1.